AMSC/CMSC 460: Midterm 1

Prof. Doron Levy

March 14, 2019

Read carefully the following instructions:

- Write your name & student ID on the exam book and sign it.
- You may not use any books, notes, or calculators.
- Solve all problems. Answer all problems after carefully reading them. Start every problem on a new page.
- Show all your work and explain everything you write.
- Exam time: 75 minutes
- Good luck!

Problems: (Each problem = 9 points)

1. Let $f(x) = x^4 - 3x^2 + 2$.

   (a) Let $P_2(x)$ be a polynomial of degree ≤ 2 that interpolates $f(x)$ at $x_0 = -1, x_1 = 0, x_2 = 1$. Write $P_2(x)$ in Lagrange form.

   (b) Let $P_3(x)$ be a polynomial of degree ≤ 3 that interpolates $f(x)$ at $x_0 = -1, x_1 = 0, x_2 = 1, x_3 = 2$. Write $P_3(x)$ in Newton form.

   (c) Let $P_4(x)$ be a polynomial of degree ≤ 4 that interpolates $f(x)$ at $x_0 = -1, x_1 = 0, x_2 = 1, x_3 = 2, x_4 = 30$. Without any calculations: what is $P_4(x)$? Justify your answer.

2. (a) Write the number 12.26 in base 2. (Compute the first 10 digits after the binary point).

   (b) Explain how 12.26 can be represented as a floating point number on a 32-bit computer.

   (c) If instead of 32 bits, the computer has 64-bit words, what would you rather increase - the number of bits representing the exponent or the number of bits representing the mantissa? Explain.

3. Let $f(x) = e^x - 3x^2 - 2$.

   (a) Find an interval that is guaranteed to include a root of $f(x)$. Justify your answer.

   (b) Write Newton's method for approximating roots of $f(x)$. Starting from $x_0 = 0$, compute the first two iterations, x_1 and x_2. Do not simplify the expression you get for x_2.

   (c) Find a function $g(x)$ for which a root of $f(x)$ is a fixed point of $g(x)$.

4. (a) Let $A = \begin{pmatrix} 9 & 3 & 6 \\ 3 & 17 & 6 \\ 6 & 6 & 30 \end{pmatrix}$. Find a Cholesky decomposition for A.

   (b) Use the decomposition from part (a), to solve $Ax = b$ with $b = \begin{pmatrix} 3 \\ 5 \\ 8 \end{pmatrix}$.

   (c) Use Gaussian elimination with scaled row pivoting to solve $Bx = c$ with $B = \begin{pmatrix} 3 & 9 & 6 \\ 9 & 3 & 6 \\ 3 & 5 & 30 \end{pmatrix}$. and $c = \begin{pmatrix} 6 \\ 6 \\ 6 \end{pmatrix}$.