# AMSC/CMSC 460: Midterm 2

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**Read carefully the following instructions:**

- Write your name & student ID on the exam book and sign it.
- You may not use any books, notes, or calculators.
- Solve all problems. Answer all problems after carefully reading them. Start every problem on a new page.
- Show all your work and explain everything you write.
- Exam time: 75 minutes
- Good luck!

**Problems: (Each problem = 10 points)**

1. (a) Assume $h > 0$. Find the most accurate approximation of $f''(x)$ using $f(x-h)$, $f(x+h)$, and $f(x+2h)$.

   (b) What is the order of accuracy of this approximation?

2. (a) Let $w(x) = \sin(x)$. Find two polynomials, $P_0(x)$ (of degree 0) and $P_1(x)$ (of degree 1) that are orthogonal with respect to $w(x)$ on $[0, \pi]$.

   (b) Normalize the polynomials you found in part (a).

   You may use: $\int x \sin(x) dx = \sin(x) - x\cos(x)$ and $\int x^2 \sin(x) dx = 2x\sin(x) + (2 - x^2)\cos(x)$.

3. Let $f(x) = x^2 + 1$. Find the weighted linear least squares approximation to $f(x)$ with respect to $w(x) = 2$ on $[-1, 1]$.

4. Find a cubic spline, $s(x)$, that interpolates

| $x$ | -1 | 0 | 1 |
|---|---|---|---|
| $y$ | 1 | 0 | 1 |

   on $[-1, 1]$ given that $s''(-1) = s''(1) = 0$. Use the interpolation points as the spline nodes.

   Note: Unfortunately you cannot solve this problem by guessing the answer. Solving it does require some calculations.