AMSC/CMSC 460: Midterm 2

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Read carefully the following instructions:

- Write your name & student ID on the exam book and sign it.
- You may not use any books, notes, or calculators.
- Solve all problems. Answer all problems after carefully reading them. Start every problem on a new page.
- Show all your work and explain everything you write.
- Exam time: 75 minutes
- Good luck!

Problems: (Each problem = 10 points)

1. (a) Write the Newton form of the interpolation polynomial of degree ≤ 3 that interpolates:

x	0	1	2	3
$f(x)$	1	0	0	0

   (b) Use the divided differences with repetitions notation to write the Hermite polynomial that interpolates the data:

$$f(0) = 1,\ f'(0) = 0,\ f(1) = 0,\ f'(1) = 0.$$

2. (a) Determine all the values of a, b, c, d, e, f for which the following function is a cubic spline

$$f(x) = \begin{cases} ax + b(x-1)^3, & x \in (-\infty, 0], \\ cx + d(x-1)^3, & x \in [0, 1], \\ ex + f(x-1)^3, & x \in [1, \infty). \end{cases}$$

   (b) Determine the values of the parameters so that the cubic spline from part (a) interpolates

x	0	1	2
$f(x)$	-2	2	7

3. (a) Let $w(x) = e^x$. Find the first two orthogonal polynomials with respect to the inner product

$$\langle f(x), g(x) \rangle_w = \int_0^1 f(x)g(x)w(x)dx.$$

   (Do not normalize the polynomials).

   (b) Normalize the polynomial of degree zero you found in part (a).

   (c) Use the orthogonal polynomials you found in part (a) to find the polynomial of degree ≤ 1, $p_1(x)$, that minimizes

$$\int_0^1 e^x (e^{-x} - p_1(x))^2 dx.$$

In solving both parts of this problem you may use the following formula:

$$\int x^n e^{ax} dx = \frac{e^{ax}}{a}\left(x^n - \frac{nx^{n-1}}{a} + \frac{n(n-1)x^{n-2}}{a^2} - \cdots \frac{(-1)^n n!}{a^n}\right) + c,\ n = \text{positive integer}$$