Example file

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1. **Newton's method.** Newton's method for finding a root of a differentiable function $f(x)$ is given by:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}. \tag{1}$$

   We note that for the formula (1) to be well-defined, we must require that $f'(x_n) \neq 0$ for any x_n. To provide us with a list of successive approximation, Newton's method (1) should be supplemented with one initial guess, say x_0. The equation (1) will then provide the values of $x_1, x_2, \ldots$

   One way of obtaining Newton's method is the following: Given a point x_n we are looking for the next point x_{n+1}. A linear approximation of $f(x)$ at x_{n+1} is

$$f(x_{n+1}) \approx f(x_n) + (x_{n+1} - x_n)f'(x_n).$$

   Since x_{n+1} should be an approximation to the root of $f(x)$, we set $f(x_{n+1}) = 0$, rearrange the terms and get (1).

2. **The secant method.** The secant method is obtained by replacing the derivative in Newton's method, $f'(x_n)$, by the following finite difference approximation:

$$f'(x_n) \approx \frac{f(x_n) - f(x_{n-1})}{x_n - x_{n-1}}. \tag{2}$$

   The secant method is thus:

$$x_{n+1} = x_n - f(x_n)\left[\frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})}\right]. \tag{3}$$

   The secant method (3) should be supplemented by two initial values, say x_0, and x_1. Using these two values, (3) will provide the values of $x_2, x_3, \ldots$.