AMSC/CMSC 460: HW #11
Due: Thursday 5/2/19 (in class)

Please submit the solution to at least one problem in LaTeX.

Note: All problems should be done as Gaussian integration.

1. Find a formula of the form

$$\int_{-\infty}^{\infty} e^{-x^2} dx \approx A_0 f(x_0) + A_1 f(x_1) + A_2 f(x_2)$$

that is exact for all polynomials of degree 5.

2. Find a formula of the form

$$\int_{0}^{\infty} f(x) e^{-x} dx \approx A_0 f(x_0) + A_1 f(x_1)$$

that is exact for all polynomials of degree 3.
Hint: Use Laguerre polynomials.

3. Find a formula of the form

$$\int_{0}^{1} f(x) x dx \approx A_0 f(x_0) + A_1 f(x_1)$$

that is exact for all polynomials of degree 3.
Hint: Start with Grahm-Schmidt process to find the orthogonal polynomials. x_0 and x_1 are the roots of the quadratic polynomial that belongs to this orthogonal family.

4. Find a formula of the form

$$\int_{0}^{1} f(x) x^2 dx \approx A_0 f(x_0) + A_1 f(x_1)$$

that is exact for all polynomials of degree 3.