AMSC/CMSC 460: HW #12
Due: Thursday 5/9/19 (in class)

For problems 1-3 please use Matlab.
The solution to these problems should include the Matlab code, and a graph of the computed solution.

1. Write a program to solve each problem on the indicated intervals using Euler's method. Each problem should be solved 3 times, with different values of h. Use $h = 1/10, 1/100, 1/1000$.

   (a)
   $$\begin{cases} x' = t + x^2, & t \in [0, 1] \\ x(0) = 1. \end{cases}$$

   (b)
   $$\begin{cases} x' = x - t, & t \in [1, 2] \\ x(1) = 1. \end{cases}$$

2. Repeat problem (1) with the Modified Euler method.

3. Use Matlab's built-in RK45 routine to solve the ODEs in problem (1).

4. Let L be an exact quantity that is approximated by $D(h)$ such that
   $$L = D(h) + a_1 h + a_3 h^3 + a_5 h^4 + \dots$$
   Use Richardson's extrapolation to obtain a third-order approximation of L. Use again Richardson's extrapolation to obtain a fourth-order approximation of L.

5. Let I be an integral that is approximated by $A(h)$ usch that
   $$I = A(h) + a_1 \sqrt{h} + a_2 h + a_3 h^{3/2} + \dots$$
   Use Romberg's integration to find a first-order aprpoximation to I. Repeat the process to find an approximation of order 3/2.