

AMSC/CMSC 460: HW #2
Due: Tuesday 2/12/19 (in class)

Please submit the solution to at least one problem in LaTeX.

1. The function

$$f(x) = \frac{x}{\sqrt{1+x^2}}$$

has a unique root $f(x) = 0$ at $x = 0$.

- (a) Show that Newton's method gives $x_{n+1} = -x_n^3$. Conclude that the method converges if and only if $|x_0| < 1$.
 - (b) Draw graphs to illustrate the first 3 iterates (x_1, x_2, x_3) when $x_0 = .25$, $x_0 = .5$, and $x_0 = 1.5$.
 - (c) Write the secant method for the same function $f(x)$. If $x_0 = 0.25$ and $x_1 = 0.35$, what is x_2 ?
2. Let p be a positive number. What is the value of the following expression?

$$x = \sqrt{p + \sqrt{p + \sqrt{p + \cdots}}}$$

Hint: observe that x can be written as the limit of a sequence for which the elements are defined as $x_{n+1} = \sqrt{p + x_n}$. Assume that x_n converges. What does it converge to? There is no need to prove convergence.

3. Let $p > 1$. What is the value of the following continued fraction?

$$x = \frac{2}{p + \frac{2}{p + \frac{2}{p + \cdots}}}$$

Hint: use the same procedure as in the previous question.

4. Write down two different fixed-point procedures for finding a zero of the function $f(x) = 3x^2 + 4\sin(x) - 2$.
5. Assume that R is a constant. If the following iterative methods converges, what will it converge to?

$$x_{n+1} = \frac{x_n(3x_n^2 + R)}{x_n^2 + 3R}.$$

6. Write a Matlab program to implement Newton's method for root finding. Use your code to run 5 iterations of Newton's method for $f(x) = x^3 - 3x^2 + x - 3$. Note that $f(3) = 0$. Plot the error in each iteration as a function of the iteration number.
7. Write a Matlab program to implement the bisection method. Use this program to compute a positive root of $f(x) = x^2 - 4x\sin x + (2\sin x)^2 - 1$. Stop when $|f(x)| < 0.01$.