Please submit the solution to at least one problem in LaTeX.

1. Use the Gram-Schmidt process to construct the first three orthogonal polynomials (polynomials of degree 0,1,2) for the following intervals and weights

   (a) $w(x) \equiv 1$, $[-2, 1]$.

   (b) $w(x) = x$, $[1, 3]$.

2. Normalize all the polynomials you found in problem #1. Remember to use the appropriate given weights and intervals.

3. Use the results of problem #1 to find the linear least squares polynomial approximation, $Q_1(x)$, to $f(x)$ for

   (a) $f(x) = x^3 - 3x - 1$, with the weight $w(x) \equiv 1$, on $[-2, 1]$

   (b) $f(x) = \frac{1}{2} \sin x + \frac{1}{4} \cos 2x$, with the weight $w(x) = x$ on $[1, 3]$

4. Use the results of problem #1 to find the quadratic least squares polynomial approximation, $Q_2(x)$, to the functions in problem #3.

5. Find the first two orthonormal polynomials (polynomials of degree 0 and degree 1) for the following weight functions $w(x)$ on the indicated intervals:

   (a) $w(x) = x^2$, $\quad 0 \le x \le 1$.

   (b) $w(x) = \sqrt{1 - x^2}$, $\quad -1 \le x \le 1$.