AMSC/CMSC 460: HW #9
Due: Thursday 4/11/19 (in class)

Please submit the solution to at least one problem in LaTeX.

1. Using the method of undetermined coefficients, establish the most accurate formula of the form

$$f'(x) \approx Af(x-h) + Bf(x+h) + Cf(x+2h) + Df(x+3h).$$

   Remember that since you have 4 unknown coefficients, it will be best to find 4 equations. The first equation will eliminate $f(x)$ from the Taylor expansion. The second equation will keep $f'(x)$. The third and fourth equations will eliminate $f''(x)$ and $f'''(x)$.

2. Using the method of undetermined coefficients, establish the most accurate formula of the form

$$f''(x) \approx Af(x) + Bf(x+h) + Cf(x+2h) + Df(x+3h).$$

   This problem is similar to problem #1. The only difference is that this time you are asked to approximate the second derivative, so make sure that you eliminate the first derivative from the expansion.

3. Using Taylor expansions, verify that the following two formulas approximate the third derivative. Find the error terms.

$$f'''(x) \approx \frac{1}{h^3}[f(x+3h) - 3f(x+2h) + 3f(x+h) - f(x)]$$

$$f'''(x) \approx \frac{1}{2h^3}[f(x+2h) - 2f(x+h) + 2f(x-h) - f(x-2h)]$$

4. Using Taylor expansions, find the error term for the approximation

$$f''(x) \approx \frac{1}{h^2}[f(x) - 2f(x+h) + f(x+2h)].$$

5. Use the values of $f(x)$ at $x-3h, x-h, x+h, x+3h$ to obtain the most accurate approximation of $f'(x)$. (This is essentially once again the method of undetermined coefficients).

6. Interpolate the values of $f(x)$ at x_0-h, x_0, x_0+2h. Use the interpolant to find an approximation for $f'(x_0 + h/2)$.

7. Interpolate the values of $f(x)$ at x_0-h, x_0, x_0+h. Use the interpolant to find an approximation for $f'(x_0 - 2h)$. Note that the approach is still valid even though this point is outside of the interval $[x_0 - h, x_0 + h]$.