

AMSC 466: Midterm 1

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Read carefully the following instructions:

- Write your name & student ID on the exam book and sign it.
- You may not use any books, notes, or calculators.
- Answer all problems after carefully reading them. Start every problem on a new page.
- Show all your work and explain everything you write.
- Exam time: 60 minutes
- Good luck!

Problems:

1. (10 points) Let $f(x) = x^4 - 1$.
- (a) Compute the Lagrange form of the interpolating polynomial $Q_2(x)$ of degree ≤ 2 that interpolates $f(x)$ at $x_0 = -1$, $x_1 = 0$, and $x_2 = 1$.
 - (b) Sketch the function $f(x)$ and the interpolant $Q_2(x)$.
 - (c) Write the error term for this interpolation, i.e., the difference $f(x) - Q_2(x)$. Assuming that $x \in [-1, 1]$, find any bound on the error term that only depends on x .

2. (10 points) Let

$$f(x) = x - \log_{10}(x + 1) - 3$$

- (a) Prove that x has at least one root in the interval $[0, 9]$.
- (b) Compute $f'(x)$. Recall that $\frac{d}{dx} \log_{10}(x) = \frac{1}{x \ln 10}$.
- (c) Using part (b), prove that $f(x)$ has only **one** root in the interval $[0, 9]$.
- (d) Write Newton's method for finding the roots of the given $f(x)$.
- (e) Denote the unique root of $f(x)$ by x^* and consider an initial value $x_0 > x^*$. Starting from any such x_0 , do you expect Newton's method to converge to the root of $f(x)$? Explain without proof. (Hint: what is $f''(x)$?)

3. (5 points)

What is the divided difference $f[1, 2, 3, 4, 5, 6, 7, 8]$, for the data

x	1	2	3	4	5	6	7	8
$f(x)$	1	2	3	4	5	6	7	8

The maximum score for this problem is 5 points if you can answer it without any computations. If you compute anything, the maximum score is 3.