AMSC 466: Midterm 1

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Read carefully the following instructions:

- Write your name & student ID on the exam book and sign it.
- You may not use any books, notes, or calculators.
- Answer all problems after carefully reading them. Start every problem on a new page.
- Show all your work and explain everything you write.
- Exam time: 60 minutes
- Good luck!

Problems:

1. (10 points. 2 points for each problem.)

   Let $f(x) = e^{2x} - 4$.

   (a) Prove that $f(x)$ must have a root in the interval $[0, 1]$. Prove that this root is unique.

   (b) Write Newton's method for finding a root of $f(x)$.

   (c) Starting from $x_0 = 3$, do you expect the method from part (b) to converge to the root of $f(x)$? Explain! If the answer is positive, what is the expected order of convergence? (In answering this question, you may use known theorems without proving them.)

   (d) Write the secant method for finding a root of $f(x)$.

   (e) If you had the choice between Newton's method and the secant method for finding the root of the given function $f(x)$, which method would you prefer to use?

2. (10 points. 2 points for each problem.)

   Let $f(x) = 8x^3 - 12x^2 + 3$.

   Let $x_0 = 0, x_1 = \frac{1}{2}, x_2 = 1$, and let $y_j = f(x_j)$ for $j = 0, 1, 2$.

   (a) Write Lagrange's form for the interpolation polynomial, $Q_2(x)$, that interpolates the data at the given (x_j, y_j), $j = 0, 1, 2$.

   (b) Write an expression for the interpolation error in the interval $[0, 1]$. Provide any upper bound on the interpolation error that is valid for any $x \in [0, 1]$.

   (c) Assume that in addition to x_0, x_1, x_2, you are given one additional interpolation point $x_3 = \frac{1}{4}$. Using the divided differences notation, write the term that should be added to the interpolation polynomial $Q_2(x)$ from part (a) in order to obtain a new polynomial, $Q_3(x)$, that interpolates the data at $x_0, \ldots, x_3$.

   (d) Without computing the divided difference in part (c), what is the coefficient of x^3 in $Q_3(x)$? Explain! If you already computed the divided difference in part (c) (something you were not asked to do), explain how the coefficient of x^3 in $Q_3(x)$ could be found directly, without this calculation.

   (e) Write Newton's form for the interpolation polynomial that interpolates the data at the given (x_j, y_j), $j = 0, 1, 2$. If you are asked to interpolate more than one function at the same interpolation points, which form would you prefer to use: Newton or Lagrange?