AMSC 466: Midterm 1

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Read carefully the following instructions:

- Write your name & student ID on the exam book and sign it.
- You may not use any books, notes, or calculators.
- Solve all problems. Answer all problems after carefully reading them. Start every problem on a new page.
- Show all your work and explain everything you write.
- Exam time: 75 minutes
- Good luck!

Problems: (Each problem = 10 points. Maximum total points = 40)

1. (a) Verify directly that for any three distinct points x_0, x_1, and x_2,

   $$f[x_0, x_1, x_2] = f[x_2, x_0, x_1].$$

   (b) Assume that x_0, x_1, x_2 are equally spaced, i.e., $x_1 = x_0 + h$ and $x_2 = x_1 + h$, with $h > 0$. Compute $f[x_0, x_1, x_2]$.

2. Consider two distinct points $x_0 < x_1$ and a twice continuously differentiable function $f(x)$. Let $Q_1(x)$ be the linear interpolant through $f(x_0)$ and $f(x_1)$.

   (a) Write the Lagrange form of $Q_1(x)$.

   (b) Using the general interpolation error formula, show that the interpolation error $|f(x) - Q_1(x)|$ in the interval $[x_0, x_1]$ is bounded by $\frac{1}{8}h^2 M$, where $h = x_1 - x_0$ and $M = \max_{x_0 \le x \le x_1} |f''(x)|$.

3. (a) Show that Newton's method for finding $\sqrt{R}$ can be written as

   $$x_{n+1} = \frac{1}{2}\left(x_n + \frac{R}{x_n}\right).$$

   (b) Let $R = AB$. Using two iterations of the formula from part (a) starting with $x_0 = A$, show that an approximation of $\sqrt{AB}$ can be written as

   $$\sqrt{AB} \approx \frac{A+B}{4} + \frac{AB}{A+B}.$$

   What happens if $x_0 = \frac{A+B}{2}$?

4. Denote the successive intervals that arise in the bisection method by $[a_0, b_0]$, $[a_1, b_1]$, etc.

   (a) Show that $a_0 \le a_1 \le a_2 \le \cdots$ and that $b_0 \ge b_1 \ge b_2 \ge \cdots$

   (b) Show that $b_n - a_n = 2^{-n}(b_0 - a_0)$.

   (c) Show that for all n, $a_n b_n + a_{n-1} b_{n-1} = a_{n-1} b_n + a_n b_{n-1}$.