

AMSC 466: Midterm 2

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**Read carefully the following instructions:**

- Write your name & student ID on the exam book and sign it.
- You may not use any books, notes, or calculators.
- Answer all problems after carefully reading them. Start every problem on a new page.
- Show all your work and explain everything you write.
- Exam time: 60 minutes
- Good luck!

### Problems:

1. (a) (5 points) Write down the conditions that should be satisfied so that the following function is a natural cubic spline on the interval  $[0, 2]$ :

$$s(x) = \begin{cases} f_1(x), & x \in [0, 1], \\ f_2(x), & x \in [1, 2]. \end{cases}$$

- (b) (5 points) Determine the values of the coefficients  $a$ ,  $b$ ,  $c$ ,  $d$ , and  $e$  so that the following  $s(x)$  is a natural cubic spline on  $[0, 2]$ :

$$s(x) = \begin{cases} 1 + x - ax^2 + bx^3, & x \in [0, 1], \\ c + d(x - 1) + e(x - 2)^2 + (x - 2)^3, & x \in [1, 2]. \end{cases}$$

2. (a) (5 points) Write the Hermite interpolation polynomial to  $f(x)$  based on the given values of  $f(a)$ ,  $f'(a)$ ,  $f(b)$ .

- (b) (5 points) Based on the result of (a), write an approximation of

$$\int_a^b f(x) dx.$$

3. (a) (5 points) Find the first two orthogonal polynomials  $p_0(x)$  and  $p_1(x)$ , with respect to the weight function  $w(x) \equiv 1$  on  $[2, 5]$ . Show your calculations.

- (b) (5 points) Find the polynomial  $Q_0(x)$  of degree zero that minimizes

$$\int_2^5 [e^{-x} - Q_0(x)]^2 dx.$$