

AMSC 466: Midterm 2

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November 15, 2011

Read carefully the following instructions:

- Write your name & student ID on the exam book and sign it.
- You may not use any books, notes, or calculators.
- Answer all problems after carefully reading them. Start every problem on a new page.
- Show all your work and explain everything you write.
- Every problem is worth 5 points. The maximum grade is 30.
- Exam time: 75 minutes
- Good luck!

Problems:

1. (a) Derive a numerical differentiation formula of order $O(h^4)$ by applying Richardson's extrapolation to

$$f'(x) = \frac{f(x+h) - f(x-h)}{2h} - \frac{h^2}{6} f'''(x) - \frac{h^4}{120} f^{(5)}(x) - \dots$$

In addition to the formula, provide the leading term in the error (i.e., the coefficient of $O(h^4)$).

- (b) What is the order of accuracy of the following approximation?

$$f'''(x) \approx \frac{1}{2h^3} [f(x+2h) - 2f(x+h) + 2f(x-h) - f(x-2h)].$$

Provide an error term for this approximation.

2. (a) Which properties of a natural cubic spline does the following function possess, and which does it not possess?

$$f(x) = \begin{cases} (x+1) + (x+1)^3, & x \in [-1, 0], \\ 4 + (x-1) + (x-1)^3, & x \in (0, 1]. \end{cases}$$

- (b) Write a linear spline on the interval $[0, 4]$, with a knot at $x = 2$, such that $f(1) = 1$, $f(3) = 0$ and $f'(0) = 1$.

3. (a) Using the Gram-Schmidt orthogonalization process, find the first two orthonormal polynomials, $p_0(x)$ and $p_1(x)$ with respect to the weight function $w(x) = 1$ on $[1, 3]$.

- (b) Find the linear least squares polynomial approximation to $f(x) = \frac{1}{x}$ on $[1, 3]$ with weight $w(x) = 1$.