AMSC 466: Midterm 2

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Read carefully the following instructions:

- Write your name & student ID on the exam book and sign it.
- You may not use any books, notes, or calculators.
- Solve all problems. Answer all problems after carefully reading them. Start every problem on a new page.
- Show all your work and explain everything you write.
- Exam time: 75 minutes
- Good luck!

1. (a) **(10 points).** Define a spline of degree k on $[a, b]$. Prove that if $S(x)$ is a spline of degree k on $[a, b]$ then $S'(x)$ is a spline of degree $k-1$ on $[a, b]$.

   (b) **(10 points).** Determine the coefficients a, b, c, d such that

$$S(x) = \begin{cases} S_0(x), & 0 \le x \le 1, \\ S_1(x), & 1 \le x \le 2, \end{cases} = \begin{cases} x^2 + x^3, & 0 \le x \le 1, \\ a + bx + cx^2 + dx^3, & 1 \le x \le 2, \end{cases}$$

   is a cubic spline that satisfies $S_1'''(x) = 12$.

2. (a) **(10 points).** Use $f(x-2h), f(x), f(x+4h)$ to write an approximation for $f''(x)$. What is the order of this approximation?

   (b) **(10 points).** What is the most accurate approximation you can write for $f'(x)$ using the same three values, $f(x-2h), f(x), f(x+4h)$? What is the order of this approximation?

3. (a) **(6 points).** Find the first two orthogonal polynomials, $P_0(x), P_1(x)$ with respect to the weight $w(x) = \sqrt{x}$ on the interval $[0, 1]$. Do not normalize them.

   (b) **(4 points).** Normalize $P_0(x)$.

   (c) **(6 points).** Let $Q_1^*(x) = a_0 P_0(x) + a_1 P_1(x)$. What should a_0, a_1 satisfy so that $Q_1^*(x)$ minimizes

$$\int_0^1 (x - Q_1(x))^2 \sqrt{x} dx.$$

   over all linear polynomials $Q_1(x)$. Express a_0 and a_1 as integrals. Do not explicitly compute these integrals quite yet.

   (d) **(4 points).** Find a_0.