MATH130
Project \#10: Absolute Extrema and Graphing

1. On an island in the Galapagos, a group of finches with beaks about 1.5 cm long eats primarily seeds that are 3-7 mm long. However, the plant producing these seeds will only reproduce in years with $>30 \mathrm{~cm}$ of rain. After a series of unusually dry years, there are no seeds $3-7 \mathrm{~mm}$ long anymore. However, plants that produce smaller seeds ( $<2$ mm long) and larger seeds ( $>1 \mathrm{~cm}$ long) are still able to reproduce in the arid conditions.
Researchers find that the survival of finches now depends on their ability to eat the larger or smaller seeds, and finches with different length beaks have different success. Their data measured a fitness term F that is dependent on the difference $D$ (in $\mathbf{~ m m}$ ) in the beak length from the average ( $\mathbf{1 . 5} \mathbf{~ c m}$ ).
Their data fit the equation: $F(D)=-0.05 D^{4}+\left(\frac{1}{10}\right) D^{3}+0.75 D^{2}$
a) D is the difference in the beak length from the average beak length of 1.5 cm . Finches were only found with beaks between 1.0 and 2.0 cm . What is the corresponding domain of D ? ( $1 \mathrm{pt)}$
b) For the domain of $D$ found in a), find all relative extrema of the function $F(D)$. Round to the hundredths place. (1pt)
c) For each relative extreme found in b), use the first or second derivative rule to determine if it is a relative minimum of relative maximum. (1pt)
d) What is the absolute minimum of $\mathrm{F}(\mathrm{D})$ within this domain? (1pt)
e) What is the absolute maximum of $\mathrm{F}(\mathrm{D})$ within this domain? (1pt)
f) F(D) represents fitness. Natural selection for a given trait, such as beak length, tends to maximize fitness for that trait. Who are the most fit among the finches? (1pt)
g) What does f) tell you about what will happen to the finch population and what seeds will be eaten most in coming years if the environment stays dry? (1pt)
h) Graph $F(D)$ on the domain found in a). Label axes, and include units, maximums, minimums and intercepts. (1pt)

i) Graph $\mathrm{F}^{\prime}(\mathrm{D})$ on the domain found in a). Label axes, and include units, maximums, minimums and intercepts. (1pt)

j) What additional information do the graphs tell you about what will happen to the finch population and what seeds will be eaten most in coming years if the environment stays dry? (1pt)
k) Bonus: What kind of natural selection will act on this finch population under these conditions? (1pt)
