Math 130 Name: Section:

Project #11: Application of extrema

When transporting animals from a controlled environment (e.g. zoo, aquarium), to be released into the wild, perhaps after rehabilitation or as a product of a captive breeding program, a minimum cage or tank size must be established so that the animal has ample space.

1. Consider a shark that requires a volume of $40ft^3$ during transport. The shark will be transported in a rectangular tank whose base length is 3 times the base width.

a. Sketch the tank and label its dimensions.

b. We want to minimize costs to build the tank. The material used to build the top and bottom (dimensions of length*width) of the tank costs $\$11/ft^2$ and the material used to build the sides and ends costs $\$5/ft^2$. Find the cost equation and simplify in terms of width and height.

c. What is the constraint, and what is its equation in terms of width and height?

d. What is the minimum cost to build the tank? (Hint: solve the constraint for one variable, plug in the solution to the cost equation, find the first derivative and then the minimum.)

e. Verify that this is the minimum cost to build the tank using the second derivative and concavity.

f. What are the dimensions (length, width, height) of the tank?

2. Consider we use a cylindrical tank instead of rectangular tank. The height of the cylindrical tank is 3 times the diameter of the base.

a. Sketch the tank and label its dimensions in terms of height and diameter.

b. The material used to build the top and bottom of the tank costs $10/ft^2$ and the material used to build the sides costs $6/ft^2$. Find the cost equation for cylindrical tank and simplify.

c. Suppose we can only afford total cost of $630, what is the constraint? How do we build the tank to maximize the tank volume? What is the maximum volume?

d. Verify that this is the maximum volume to build the tank using the second derivative and concavity.