Math 130 Section#__________ Name

Project #12: Antiderivatives

So far this semester, we have been determining the rates of change for a given biological function, usually one that describes how a quantity changes over time. However, sometimes it may be difficult or impossible to determine the absolute quantity of a function, and instead the rate at which the quantity changes with respect to time is known. In this case, we can work backwards to determine the relationship between quantity and time by using the known rate of change. This process is called antidifferentiation, and is done by reversing the rules we learned for differentiation. For example, the function $f(x) = x^2$ has the derivative $f'(x) = 2x$. If we began with knowing that $f'(x)=2x$, we could work backwards to determine that $f(x)= x^2$. However, in the reverse direction, there is a constant problem. Since the derivative of a constant is zero, the antiderivative of any function must include a correction for the unknown constant. Thus, the antiderivative of $f'(x) = 2x$ becomes $F(x) = x^2 + C$, where C represents the unknown constant.

1. A bacterial population in a flask experiences a growth rate that can be described by the equation

$$R'(t) = 3t^2 - 36t + 96$$

where t is the time in hours, and the equation is valid over the domain [0,8].

a. Given this equation, at what time is the population maximized? Confirm by using the second derivative test for concavity. (2points)

b. What is the antiderivative, R(t) for this equation? (1point)

c. At the population maximum, the population size is 1160 million. Using this information, solve for the constant, C, and write the complete equation that describes the relationship between the number of bacteria (in millions) with time (in hours). (2points)

d. What is the population size at 8 hours? (1point)

2. In 2009, Gill and his colleague have provided direct evidence that a shorebird, the Alaskan bar-tailed godwit, makes its eight-day autumn migration from Alaska to New Zealand in one step, with no stopovers to rest or refuel.

The minimum requirement for any long flight is that enough fuel is taken on board before departure to sustain the bird for the duration of the flight; in the godwit's case this is about 200 hours. Assuming that the rate of fuel consumption is a fixed proportion of the migration time, the velocity of the migration is proportion to the fuel consumption, which could be roughly represented by this equation: V (velocity)=110-0.5t. (Unit of V: km/hour)

a. The distance the godwit be able to migrate is the antiderivative of V. What is the antiderivative of V? (1 point)

b. Find the value of C, if godwits are able to migrate 11,000km in 200 hours. (1 point)

c. Plot Velocity vs. time. What is the velocity at t= 200 hours? (2 points)