## Biomodule 9: Sections 5.3 and 5.4

Human hemoglobin is made of four globular protein subunits, each of which binds an oxygen molecule. The binding of oxygen can be represented as $Y_{O_{2}}=\frac{p^{4}}{c^{4}+p^{4}}$ where $Y_{O_{2}}$ represents the saturation of the hemoglobin molecule with oxygen (from 0 to 1 ), $p$ is the partial pressure of oxygen in the blood in mmHg , and $c$ is a constant that defines the binding ability of a given hemoglobin molecule. The subunits of hemoglobin have cooperative binding, meaning that the rate of oxygen binding for the entire molecule changes as each subunit binds an oxygen. The typical atmospheric partial pressure of oxygen is 160 mmHg , however the partial pressure of oxygen in a typical human lung is only 47 mmHg .

1) Graph the saturation of hemoglobin in reference to the partial pressure of oxygen in the blood for the following hemoglobin molecules (on the same graph): (2pts)

Standard human hemoglobin: $\quad c=26.5 \mathrm{mmHg}$
Fetal hemoglobin:
c $=20 \mathrm{mmHg}$
Sickle cell anemia hemoglobin: $\quad c=34 \mathrm{mmHg}$

2) Find the point of inflection for each type of hemoglobin. (Hint: if you use " $c$ " as the constant while deriving your functions, you can just substitute the values in for c at the end.) (2pts)
3) The affinity of hemoglobin for oxygen can be described as the tendency of a hemoglobin molecule to bind an oxygen molecule based on a given partial pressure of oxygen (where higher affinity means more likely to bind oxygen). Based on your graph and your answer to 2, which hemoglobin molecule has the highest affinity? The lowest? Explain your reasoning. (1pt)

Bees must fly long distances between flowers. Since each bee wants to bring back as much nectar as possible over the course of the day, she needs to maximize the rate of nectar uptake per visit (in microliters/minute), which includes the travel time (in minutes) between flowers.

Rate at which nectar is collected $=\frac{\text { food per visit }}{\text { total time per visit }}$ OR
$R\left(T_{f}\right)=\frac{\text { food per visit }\left[F\left(T_{f}\right)\right]}{\text { time on flower }\left(T_{f}\right)+\text { travel time }\left(T_{t}\right)}$ OR
$R\left(T_{f}\right)=\frac{F\left(T_{f}\right)}{\left(T_{f}+T_{t}\right)}$
where $F\left(T_{f}\right)=\frac{T_{f}}{T_{f}+0.8}$ and $\mathrm{T}_{\mathrm{t}}=1$ minute
4) Rewrite the equation for $\mathbf{R}\left(\mathbf{T}_{\mathbf{f}}\right)$ using the information given, and simplify. (1pt)
5) Find the first derivative, $\mathbf{R}^{\prime}\left(\mathbf{T}_{f}\right)$. Determine where the original function $\mathbf{R}\left(\mathbf{T}_{f}\right)$ is increasing and decreasing on the domain [0,3]. Use this information to find the values of any relative extrema. (2pts)
6) Graph $\mathbf{R}\left(\mathbf{T}_{\mathbf{f}}\right)$ and $\mathbf{R}^{\prime}\left(\mathbf{T}_{\mathbf{f}}\right)$ on the domain [0,3]. (1pt)

7) Now how many minutes should the bee spend on each flower? Interpret both graphs in terms of the bee maximizing its time at each flower. (1pt)

