Stat 410: Midterm 2

Prof. Doron Levy

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Read carefully the following instructions:

- Write your name & student ID on the exam book and sign it.

- You may <u>not</u> use any books, notes, or calculators.

- Solve all problems. Answer all problems after carefully reading them. Start every problem on a new page.

- Show all your work and explain everything you write.

- Exam time: 75 minutes

- Good luck!

Problems: (Each problem $= 10$ points. Maximum total points $= 60$)

1. Suppose that the average number of bicycles stolen weekly in College Park is 3.6. Approximate the probability that there will be

   (a) No stolen bicycles in the next two weeks.

   (b) At least 2 stolen bicycles in the next week.

2. Assume a random variable with $\mathrm{E}[X] = 2$ and $\mathrm{Var}[X] = 3$.

   (a) Compute $\mathrm{Var}[-2 - 5X]$.

   (b) Compute $\mathrm{E}[(2 + X)^2]$

3. Let X be a continuous random variable with the density

$$f_X(x) = \begin{cases} ce^{-2x}, & x \geq 1, \\ 0, & x < 1. \end{cases}$$

   (a) Find the constant c such that $f_X(x)$ is indeed a probability density function.

   (b) Compute, $F_X(x)$, the cumulative distribution function of X.

   (c) Find $P(X \geq 2)$.

   (d) Find $\mathrm{E}[X]$ and $\mathrm{Var}[X]$. You may use the formula (for n positive integer):

$$\int x^n e^{ax} dx = \frac{e^{ax}}{a} \left(x^n - \frac{nx^{n-1}}{a} + \frac{n(n-1)x^{n-2}}{a^2} - \cdots \frac{(-1)^n n!}{a^n} \right).$$

4. Resistors produced by a company will be defective with probability 0.05, independently of one another. The company sells the resistors in packages of 40 and offers a money-back guarantee that at most 1 of the 40 resistors is defective. What is the probability that a package of resistors will be replaced?

5. When coin #1 is flipped, it lands on heads with probability 0.3; when coin #2 is flipped, it lands on heads with probability 0.6. One of these coins is randomly chosen and flipped 12 times.

   (a) What is the probability that the coin lands on heads on exactly 8 of the 12 flips?

   (b) Given that the first two flips out of these 12 flips lands heads, what is the conditional probability that exactly 8 of the 12 flips land on heads?

6. If independent trials, each resulting in a success with probability p, are performed, what is the probability of r successes occurring before m failures?