Joint Distribution.

1. Let $(X, Y)$ have density $p(x, y) = 2y^2 e^{-xy}$ for $x \geq 0, 0 \leq y \leq 1$ and 0 otherwise. Find marginal distributions of $X$ and $Y$.

2. Let $(X, Y)$ be uniformly distributed on $[0, 1]^2$. Find the joint distribution of $\max(X, Y)$ and $|X - Y|$.

3. A box contains 5 red, 10 blue and 15 green balls. 2 balls are chosen at random (a) with replacement; (b) without replacement. Find the joint distribution of the number of red and blue balls as well as their marginal distributions.

4. Let $X_1, X_2 \ldots X_n$ be uniformly distributed on the disc

$$X_1^2 + X_2^2 + \ldots X_n^2 = n.$$ 

If $n$ is large, approximate the marginal distribution of $X_1$.

5. Let $(X, Y)$ be uniformly distributed in the unit circle. Find the probability that

(a) $(X, Y)$ is within distance $1/2$ from the origin.

(b) $X$ and $Y$ are both positive and $Y/X < 2$.

6. In a certain state 50% of all cars are American, 30% Japanese and 20% European. 20 cars from that state passed a tall booth.

(a) Find a joint distribution of the number of American and number of Japanese cars.

(b) Find marginal distribution of the number of American cars.

(c) Find the distribution of the number of American cars given that only 2 cars were European.

7. Let $(X, Y)$ have continuous distribution with smooth density. Suppose that $X$ and $Y$ are independent and also that $X + Y$ and $X - Y$ are independent. Find the distributions of $X, Y$.

8. Let $X_1, X_2, \ldots X_{10}$ be independent uniformly distributed on $[0, 1]$. Find the joint distribution of their maximum and minimum.

9. Find the distribution of $X + Y$ where $X$ and $Y$ are independent and

(a) $p_X(x) = x^{s-1} e^{-x}/\Gamma(s)$, $p_Y(y) = x^{t-1} e^{-x}/\Gamma(t)$.

(b) $X \sim N(1, 2)$, $Y \sim N(3, 4)$.

(c) $X \sim \text{Bin}(10, 1/3)$, $Y \sim \text{Bin}(2, 1/3)$

(d) $X \sim \text{P}(1)$, $Y \sim \text{P}(2)$.

10. Let $X$ and $Y$ be independent having exponential distribution with parameter 1. Find the joint distribution of $X^2$ and $X + Y$. 

1
11. Let $X_1, X_2, X_3, X_4$ denote the independent random variables having exponential distribution with parameter 1. Find the probability that

$$\max(X_1, X_2, X_3, X_4) - \min(X_1, X_2, X_3, X_4) < 1.$$