The topics for midterm 1.

(1) Basic properties of probability.

$$P(A) = 1 - P(A').$$

$$P(A \bigcup B) = P(A) + P(B) - P(A \bigcap B).$$

$$P(A \bigcup B \bigcup C) = P(A)+P(B)+P(C)-P(A \bigcap B)-P(A \bigcap C)-P(B \bigcap C)+P(A \bigcap B \bigcap C).$$

(2) Vann diagrams.

(3) Equally likely events

$$P(A) = \frac{\#(\text{outcomes in } A)}{\#(\text{outcomes in } S)}$$

(4) Product rule

$$N = n_1 n_2.$$

(5) Permutations of size k of n objects.

$$P_{k,n} = \frac{n!}{(n-k)!}.$$

(6) Combinations of size k of n objects.

$$\left(\begin{array}{c} n \\ k \end{array} \right) = \frac{n!}{k!(n-k)!}.$$

(7) Tree diagrams.

(8) Conditional probability

$$P(A|B) = \frac{P(A \bigcap B)}{P(B)}$$

(9) Multiplication rule

$$P(A \bigcap B) = P(B)P(A|B).$$

(10) Law of Total probability

$$P(B) = P(A_1)P(B|A_1) + P(A_2)P(B|A_2) + \cdots + P(A_k)P(B|A_k).$$

(11) Bayes' Formula

$$P(A_j|B) = \frac{P(A_j)P(B|A_j)}{P(A_1)P(B|A_1) + P(A_2)P(B|A_2) + \cdots + P(A_k)P(B|A_k)}.$$

(12) Independent Events

$$P(A_1 \bigcap A_2) = P(A_1)P(A_2).$$

(13) Distribution Function and Cumulative Distribution Function

$$F(b) = \sum_{x \leq b} p(x),$$

$$P(a \leq X \leq b) = F(b) - F(a-).$$

(14) Expectation

$$E(X) = \sum_{S} X(s)p(s) = \sum_{x} xp(x).$$

(15) Properties of expectation

$$E(cX) = cE(X), \quad E(X_1 + X_2) = E(X_1) + E(X_2), \quad E(c) = c.$$

(16) Variance and standard deviation

$$V(x) = E((X - E(X))^2) = E(X^2) - (E(X))^2, \quad \sigma_X = \sqrt{V(X)}.$$

(17) Properties of variance

$$V(cX) = c^2 V(X), \quad V(X + b) = V(X).$$