

### Discrete distributions.

1. A die is rolled 5 times. Let  $X$  be the number 6 appears. Find  $P(X = 2)$ ,  $P(X = 4)$ .
2. A coin is tossed 10 times. Find the probability of 6 heads.
3. 40 % of all customers of Fried Crocodile Restaurant order crocodile steak. During a certain evening 25 people had dinner. What is the probability that the number of steaks ordered is
  - (a) exactly 10
  - (b) less than 10
  - (c) between 10 and 15 (inclusive).
4. An urn contains 3 red balls and 5 blue ones. A ball is chosen randomly. If this ball is red the process stops, otherwise the ball is returned to the urn and the process continues. Let  $X$  be the number of blue balls drawn before the process stops. Compute
  - (a)  $P(X = 2)$
  - (b)  $P(2 \leq X \leq 6)$
  - (c)  $P(4 \leq X \leq 8 | X \geq 2)$
5. A family decides to have children until it has a boy. Let  $X$  be the number of girls in the family.
  - (a) Compute  $E(X)$ ,  $V(X)$ .
  - (b) Do the same for three boys.
6. An urn contains 3 red balls and 5 blue ones. 5 balls are chosen without replacement. Let  $X$  be the number of blue balls. Compute
  - (a)  $P(X = 2)$
  - (b)  $EX$  and  $VX$ .
7. For each of the examples below identify if the variable of interest has binomial, negative binomial, hypergeometric distribution or neither.
  - (a) An exam has 20 questions. John knows how to answer 7. He will be asked 4 questions. Let  $U$  be the number of questions he knows how to answer.
  - (b) John has 5 exams. He estimates that for each exam he has 30% chance of getting A, 40% of getting B and 30% of getting C independently of other exams. Let  $X$  be the number of exams he gets A. Let  $Y$  be the number of exams John gets A or B.
  - (c) Do part (a) if John has 30% chance of getting A, 40% of getting B and 30% of getting C on his 2 math exams and 50% chance of getting A, 20% of getting B and 30% of getting C for 3 non math exams.
  - (d) John takes a certain test until he gets a score of 80. He estimates that each try he has 20% chance of getting that score independently of other tries. Let  $Z$  be the number of failures.
  - (e) John buys cereal. Each box has a card of either dog, cat, bird or fish. When John collects all 4 types he can get a free box. Let  $V$  be the number of boxes John has to buy before he has all 4 types of cards.
8. Mary wants to beat the world jumping record. During home training she can do it in  $1/3$  of all jumps. Assume that the results of consecutive jumps are independent. She has

3 attempts at each jumping competition. What is the probability that she fails in the first competition and succeeds in the second?

9. The probability to win in a lottery is  $1/1000$ . Joe plays 50 times a year for 30 years.

(a) What is the probability he never wins? wins once? twice? three times?

(b) What is the probability that Joe will win nothing during the first 15 years and wins twice during the next 15 years?

10. A repair office is open from 8 till 4. During that time requests form Poisson process with average 1 request every 4 hours. Joe comes every 2 hours to check new requests.

(a) What is the probability that during a certain day he will get  $(0,1,2,1)$  requests at 10, noon, 2 and 4 respectively?

(b) Suppose it takes Joe 1 hour to serve each request. If after checking the office Joe has nothing to do, he drinks a glass of beer. If he had no carryovers from the previous day, what is the probability that he is able to drink at noon?

(c) Let  $X$  be the number of requests during a particular day. Compute  $E(X)$  and  $V(X)$ .