Continuous distributions.

1. *Leaks if a water pipe form a Poisson process with 4 leaks/year on avarage.*

(a) Compute the probability that during the first 6 months there will be no leaks, in the next 6 months there will be three leaks and during the second year there will be 3 leaks.

(b) Let T *be the time of the first leak. Compute* $P(T \leq t)$.

2. *Suppose that* X *has probability distribution function equal to* cx^2, *if* $x \in [0, 1]$ *and equal to* 0 *otherwise.*

(a) Find c. (b) Compute the cumulative distribution function. (c) Compute EX *and* VX. *(d) Compute median, 25th and 75th percentiles.*

3. *Let* X *have the cumulative distribution function* $F(x) = \frac{x+x^3}{2}$ *if* $0 \leq x \leq 1$, 0 *if* $x \leq 0$ *and* 1 *if* $x \geq 1$. *Compute*

(a) probability density function (b) EX (c) VX.

4. *A passanger arrives at the bus stop. His waiting time* T *has uniform distribution on* $[0, 10]$.

(a) Compute the cumulative distribution function.

(b) Compute $P(2 \leq T \leq 5)$, $P(T > 3)$.

(c) Compute ET *and* VT.

5. *Suppose that a random variable* X *has proability density function* $p(x) = e^{-x}$ *if* $x > 0$ *and* 0 *otherwise. Find the median of* X.

6. *Let* Z *be standard normal random variable. Find its median, 25th and 75th percentile.*

7. *The weight of a box of appples has normal distribution with mean 30 lbs and standard deviation 2 lbs. Compute the probability that the box weights*

(a) exactly 31 pounds; (b) between 30 and 32 pounds (c) less than 29 pounds.

8. *A coin is tossed 100 times compute approximately the probability that*

(a) there are exactly 50 heads. (b) the number of heads is between 47 and 52 (inclusive).

9. *Lifetime* T *of a certain component has exponetial distribution with mean 10 years.*

(a) Compute the probability that $T > 5$, $T > 10$, $T > 15$.

(b) Compute the probability that componewnt will work for 15 years or more given that it worked for 5 years.

(c) Let S *be the lifetime of the component measured in decades, thus* $S = T/10$. *Compute the distribution of* S.

10. *In problem 1 compute let* T_2 *and* T_3 *be the times of the second and third leak respectively. Find their probability density functions.*

11. *Let* Z *be the standard normal random variable and* $X = Z^2$. *Find the distribution of* X.

12. *A malfunctioning of a certain engine could be caused by two components. Lifetimes of those components are independent and have exponential distributions with means 2 and 3 years respectively. Find the average lifetime of the engine.*