Joint distribution.

1. *John plays chess with Bill and Bob. For each game he gets 1 point for a win, 1/2 point for a draw and 0 points for a loss. Let X be outcome of his game with Bill and Y be outcome of his game with Bob. Suppose that the joint distribution of X and Y is given in the following table.*

$Y \backslash X$	0	1/2	1
0	.10	.07	.11
1/2	.07	.05	.10
1	.05	.10	.35

(a) Compute the marginal distributions of X and Y;
(b) Compute the probability that both games have the same result; that John losses at least once.
(c) Find the distribution of $X+Y$.
(d) Find the distribution of Y given that $X = \frac{1}{2}$.

2. *Suppose 50 % of all drivers have American cars, 40 % have Japanese cars and 10 % have European cars. Consider 15 consecutive cars crossing certain intersection.*
(a) What is the probability that 8 are American, 5 are Japanese and 2 are European; 9 American and 6 Japanese ?
(b) Find the marginal distribution of the number of American cars.

3. *Let (X,Y) have density $p(x,y) = k(2x+y)$ if $0 \le x \le 1$, $0 \le y \le 1$ and 0 otherwise.*
(a) Find the constant k.
(b) Compute $P(X > Y)$.
(c) Find the marginal distribution of Y.
(d) Find the distribution of Y given that $X = \frac{1}{2}$.
(e) Compute $E(X^2)$.
(f) Compute VX, VY and $Cov(X,Y)$.

4. *Suppose that Johns arrival time to a bus stop is uniform on the segment 1:00 to 1:10 and bus arrival time is uniform on the segment 1:00 to 1:20 and is independent of John's. What is the probability that John misses the bus; that he has to wait more than 5 min?*

5. *Let X and Y be independent, $X \sim Exp(1)$, $Y \sim Exp(2)$. Compute $P(X > Y)$.*

6. *X and Y are independent. $Z = X+Y$. Find the distribution of Z if*
(a) $X \sim Pois(2)$, $Y \sim Pois(3)$
(b) $X \sim Uni(0,1)$, $Y \sim Uni(0,1)$;
(c) $X \sim Exp(5)$, $Y \sim Exp(2)$.

7. *X_1, X_2, X_3 and X_4 are independent and uniformly distributed on $[0,1]$. Let $M = \max(X_1, X_2, X_3, X_4)$. Find the distribution of M.*

8. *Let (X,Y) be uniformly distributed in a triangle $x \ge 0$, $y \ge 0$, $2x+3y \le 6$.*
(a) Find marginal distribution of X. (b) Compute $P(X > Y)$. (c) Compute VX, VY and $Cov(X,Y)$.