(1) Let $X$ have geometric distribution with parameter $\frac{2}{3}$, that is $P(X = k) = \frac{2}{3} \left(\frac{1}{3}\right)^{k-1}$. Find
   
(a) $P(5 < X < 10)$.

(b) $E(2^X)$.

(2) There are three stocks whose price at present is $100 each. Their prices after 1 year are independent random variable with means $120 and variances 100, 200 and 300 respectively. How should you distribute $1000 between stocks 1, 2 and 3 to minimize the variance of you portfolio value after 1 year?

(3) If the harvest is good the price of the plum producer stock is going to have uniform distribution between $110 to $119 (that is $P(X = k) = \frac{1}{10}$) while if the harvest is bad the price of the plum producer stock is going to have uniform distribution between $96 and $115 (that is $P(X = k) = \frac{1}{20}$). The probability that the harvest is good is $\frac{3}{4}$. Find the expected price of the stock.

(4) Find the rate of return on the following investment. You pay immediately $1000 and get $500 after 5 years and $1000 after 10 years. You can find the answer either analytically or numerically.

(5) If the interest rate is 10 % per period which flow of payments is better for you to receive:
   
(a) $1000 for the first 10 periods and $2000 for the next 10 periods;

(b) $2000 for the first 10 periods and $1000 for the next 10 periods;

(c) $1700 for 20 periods?

(6) You borrow $100000 from the bank. The interest rate is 0.4 % per month for the first 10 years and 0.6 % per month for the next 10 years.
   
(a) Which fixed monthly payment will allow you to repay the loan after 20 years?

(b) Find the effective interest rate of this loan.

(7) A certain stock cost now $100 and after one period it will cost either $200 or $50. The interest rate is 10 %. Find the price of the option which pays you $10 if the price is $200.

(8) The following tables show the dependence of the price $C$ of the call options for of certain stocks on the strike price $K$. Suppose that the exercise time is 1 year and the interest rate is 10 % per year. In each case identify the arbitrage.

(a) $K$ 110 120 130
   $C$ 20 18 15

(b) $K$ 110 120 130
   $C$ 40 30 25

(9) Both stock 1 and stock 2 cost $100 at present. After 1 year stock 1 will cost either $100 or $150 and stock 1 will cost either $100 or $200. Let $C_1$ be the price of the call option on stock 1 with exercise time 1 year and strike price $120 and $C_2$ be the price of the call option on stock 2 with exercise time 1 year and strike price $120. The interest rate is 5 % per year.
   
(a) Compute $C_1$ and $C_2$.

(b) Stock 3 costs $100 at present and it will cost either $100, or $150 or $200 after 1 year. Let $C_3$ be the price of the call option on stock 3 with exercise time 1 year and strike price $120. Use part (a) to estimate $C_3$.

(10) There are two investments whose present value under three different circumstances is given in the tables below. In which of the cases is arbitrage possible:

(a) $S_1$ -5 2 2
   $S_2$ 0 2 -2

(b) $S_1$ -1 2 2
   $S_2$ 5 2 -2

(c) $S_1$ 6 2 -2
   $S_2$ 2 -3 -2

(11) A certain stock cost now $100. After the first period the cost will either go up 20 % or down 10 %. After the first period the cost will either go up 20 % or stay the same as before. The interest rate is 5 %. Find the price of the call option with exercise time 2 periods and strike price $110.

(12) Consider multi period binomial model in which stock can go up 10 % or down % 20. Find the price of the option which after the third period allows you to sell the stock for its highest price up to that time.