STAT400. Sample questions for midterm 2.

1. *In this problem you may neglect the probability of twins.*

(a) A family has 5 children. Find the probability that they have 2 boys and 3 girls.

(b) A family decides to have children until they have 3 girls. Find the probability that they have 2 boys.

(c) A family decides to have children until they have 3 girls. Let C be the total number of children in the family. Compute EC and VC.

2. *(a) 40% of lightbulb produced by Shining Beauty company have suboptimal performance. 5 bulbs are chosen for test. Find the probability that 2 bulbs or less have suboptimal performance.*

(b) 40% of lightbulb produced by Shining Beauty company have suboptimal performance. 500 bulbs are chosen for test. Compute approximately the probability that 210 bulbs or less have suboptimal performance.

(c) 0.4% of lightbulb produced by Light company have suboptimal performance. 500 bulbs are chosen for test. Compute approximately the probability that 2 bulbs or less have suboptimal performance.

3. *Calls to a customer service center form Poisson process with intensity 4 calls per hour.*

(a) Find the probability that there are less than 3 calls between 9:00 and 10:00.

(b) John works from 9:00 am to 3:00 pm. His shift is divided into 6 one hour intervals. John calls an interval easy if there are less than 3 calls during that interval. Find the probability that during a particular day he has at least one easy interval.

(c) Find the probability that the first call during John watch arrives between 9:00 and 9:20 and the second between 9:20 and 9:40.

4. *Let X have cumultive density function*

$$F(x) = \begin{cases} 0, & \text{if } x \leq 0 \\ \frac{x^2+x^4}{2} & \text{if } 0 \leq x \leq 1 \\ 1 & \text{if } x \geq 0 \end{cases} .$$

(a) Compute density of X.

(b) Compute EX and VX.

(c) Let $Y = X^3$. Compute EY and VY.

5. *Let X have normal distribution with mean 2 and standard deviation 2.*

(a) Compute 25 and 75th percentiles.

(b) Compute $P(X > 5)$.

(c) Compute $E(X^2)$.

6. *Let the distribution of X and Y be given in the following table.*

$X\backslash Y$	1	2	3
0	.05	.10	15
1	.05	.05	.10
2	.20	.05	.25

(a) Compute the marginal distributions of X and Y.
(b) Compute $P(X = Y)$.
(c) Compute $Cov(X, Y)$.

7. *Let (X, Y) have uniform distribution on the trapezoid $0 \le y \le 1$, $0 \le x \le 1 + y$.*
(a) Compute the marginal distributions of X and Y.
(b) Compute $V(X)$.
(c) Compute $Cov(X, Y)$.

8. *Let X be independent, X have uniform distribution on $[0, 1]$ and Y have exponential distribution with parameters 1. Let $Z = X + Y$.*
(a) Compute the density of Z.
(b) Compute $Cov(X, Z)$.
(c) Compute $P(3Y > Z)$.

9. *Let $S = X_1 + X_2 + \dots X_{162}$ where X_j are independent identically distributed random variables. Suppose that X_j have density equal to $2x$ if $0 \le x \le 1$ and equal to 0 otherwise.*
(a) Compute ES.
(b) Compute VS.
(c) Compute approximately $P(S > 110)$.

10. *60 scientists from 30 universities attend a conference. The conference includes a lunch in a cafetria which has 30 tables each suitable for 2 people. The people seated for lunch at random. Let $X_j = 1$ if the scientists from university j seat togather and $X_j = 0$ otherwise.*
(a) Compute $E(X_1)$ and $V(X_1)$.
(b) Compute $Cov(X_1, X_2)$.
(c) Let $X = X_1 + X_2 + \dots X_{30}$ be the number of tables occupied by the people from the same university. Compute EX and VX.