Friday March 30

3:15. Department Colloquim Hakan Eliasson *Birkhoff Normal Form and a problem of Herman*

Saturday March 31

9:00–9:10 Introductory remarks.

9:10–10:05 Michal Misiurewicz *Random interval homeomorphisms*

10:05–10:25 Coffee break

10:25–11:20 Francesco Cellarosi *On the Ergodic Properties of Square-Free Numbers*

11:30–12:20 Yakov Sinai *Binary Correlations of the Moebius function*

12:20–2:20 lunch

Special session celebrating James Yorke 70th birthday

2:20–3:15 James Yorke *The many facets of chaos*

3:15–3:40 coffee break

3:40-4:35 Brian Hunt *Lyapunov dimension for skew-product systems*

4:50-5:45 Vadim Kaloshin *Arnold diffusion via Normally Hyperbolic Invariant Cylinders*

7:00 Banquet at *Chef's Secret*

Sunday April 1

Session on dynamics and PDEs

9:00–9:55 Alexander Shnirelman *On the shooting problem for the Euler equations*

9:55–10:20 Coffee break

10:20–11:15 Lai-Sang Young *Toward a smooth ergodic theory for infinite dimensional systems*

11:30–12:20 Steve Zelditch *Paley-Wiener, nodal sets and ergodicity on analytic Riemannian manifolds*

12:20–2:20 lunch

2:20–3:15 Anthony Quas *Semi-invertible multiplicative ergodic theorems and applications*

3:15–3:40 coffee break

3:40-4:10 Parallel session I

Ben Webb *Structural Transformations and Improved Stability Estimates of Dynamcial Networks*

AND

Kiran Parkhe (Northwestern) *Distortion for diffeomorphisms of surfaces with boundary*

4:20–4:50 Parallel session II

Jianyu Chen *Coexistence of zero and nonzero Lyapunov exponents*

AND

Ilya Vinogradov (Princeton) *Effective bisector estimate for $PSL(2,\mathbb{C})$ with applications to circle packings*

5:00-5:55 Carlos Mateus *On the central Oseledets bundle of the Kontsevich-Zorich cocycle*

Monday April 2.

HAMILTONIAN DYNAMICS DAY

9:00–9:55 Rafael de la Llave *Systems with local interactions: localized quasi-periodic and almost periodic breathers*

9:55–10:20 Coffee break

10:20–11:15 Mark Levi *A variational approach to time–dependent billiards*

11:30–12:20 Vadim Zharnitsky *Hamiltonian dynamics in the linear search problem*

12:20–2:20 lunch

2:15–3:10 Nandor Simanyi *Transversality of Singularities and Non-hyperbolicity Manifolds*

3:20- 3:50 Parallel session III

Alexey Korepanov *Nonequilibrium stationary states in the thermostatted Lorentz gas*

AND

TBA

3:50–4:10 coffee break

4:10-5:05 Jacopo de Simoi *Cyclicity one elliptic islands for the standard family*

5:20-6:15 Konstantin Khanin *On the renormalization conjecture for circle homeomorphisms with breaks*

Tuesday April 3.

9:00–9:55 Maciej Zworski *Resonance free strips for normally hyperbolic dynamics*

9:55-10:15 coffee break

10:15-11:10 Yingfei Yi *Noise Perturbation In Finite Dimension*

11:20-12:15 Sheldon Newhouse *Chaos for many parameters in the Lorenz system*

Abstracts.

Invited talks.

Francesco Cellarosi *On the Ergodic Properties of Square-Free Numbers*

We study binary and multiple correlations for the set of square-free numbers and we construct a dynamical systems naturally associated to them. We prove that this dynamical system has pure point spectrum and it is therefore isomorphic to a translation on a compact abelian group. In particular, the system is ergodic but not weakly mixing, and it has zero metric entropy. The latter results were recently announced by Peter Sarnak, and we provide an alternative approach. Joint work with Yakov Sinai.

Rafael de la Llave *Systems with local interactions: localized quasi-periodic and almost periodic breathers.*

We consider Hamiltonian systems with a localized coupling. For example, pendula coupled by springs. We will show that, under rather general assumptions, there are solutions that are several sites oscillating quasi-periodically. (the set of sites that oscillate may be unbounded and the number of independent frequency finite). This is joint work with E. Fontich, and Y. Sire.

We will also show that these solutions admit stable and unstable manifolds. (joint work with D. Blazevski).

The main tool is a functional analysis description of objects that are localized. This also has applications to the ergodic theory of hyperbolic coupled maps, previously obtained with M. Jiang and with E. Fontich and P. Martin.

Jacopo De Simoi *Cyclicity one elliptic islands for the standard family*

We study abundance of a special class of elliptic islands for the standard family of area preserving diffeomorphism for large parameter values, i.e. far from the KAM regime. Outside a bounded set of parameter values, we prove that the measure of the set of parameter values for which an infinite number of such elliptic islands coexist is zero and that Carleson conjecture holds when restricted to this specific class of elliptic islands. On the other hand we construct a residual set of large parameter values of positive Hausdorff dimension for which the associated standard map admits infinitely many elliptic islands whose centers accumulate on a locally maximal hyperbolic set.

Hakan Eliasson *Birkhoff Normal Form and a problem of Herman*

We shall discuss some questions related to the Birkhoff Normal Form of an analytic Hamiltonian system near a Diophantine equilibrium. This is a classical subject but several questions still remain open.

We shall in particular report on a recent work (joint with B. Fayad and R. Krikorian) which is related to (but does not solve) a question raised by M. Herman in his ICM lecture in Berlin in 1996: is such an equilibrium always accumulated by a set of KAM tori of positive Lebesgue measure?

Brian Hunt *Lyapunov dimension for skew-product systems*

(This is joint work with Maik Groger.) Kaplan and Yorke conjectured that "typical" attractors of smooth dynamical systems have a physical measure for which the Lyapunov dimension (defined in terms of Lyapunov exponents) is equal to a geometrically defined dimension (now understood to be information dimension or equivalent). We show for a class of skew-product systems (a pair of baker's maps with one-way coupling) that for some parameter values the Kaplan-Yorke equality holds for a prevalent (measure-theoretically typical) set of coupling functions, whereas for other parameter values the Kaplan-Yorke equality fails for all coupling functions. We discuss modification of the Lyapunov dimension for skew-product systems to agree more generally with geometric attractor dimension.

Vadim Kaloshin *Arnold diffusion via Normally Hyperbolic Invariant Cylinders*

In 1964 Arnold constructed an example of instabilities for nearly integrable systems and conjectured that generically this phenomenon takes place. There has been some progress attacking this conjecture in the past decade. Jointly with Ke Zhang we present a new approach to solve this problem. It is based on a construction of crumpled and flower Normally Hyperbolic Invariant Cylinders with kissing property. Then to construct diffusion along these cylinders we apply Mather variational mechanism. A part of the project is also joint with P. Bernard.

Carlos Matheus *On the central Oseledets bundle of the Kontsevich-Zorich cocycle*

The Lyapunov exponents of the Kontsevich-Zorich (KZ) cocycle are important quantities in the study of the deviations of ergodic averages of interval exchange transformations and translation flows.

In this talk, we will be interested in mechanisms leading to the presence of zero Lyapunov exponents of KZ cocycles, and hence a non-trivial central Oseledets bundle.

Following some works in collaboration with G. Forni and A. Zorich, we will discuss the relationship between the central Oseledets bundle and the annihilator of a certain second fundamental form (giving the first variation along Teichmuller geodesics), and we will see a class of examples where these two objects coincide (that is, a class whose the central Oseledets bundle has a "clear geometric explanation"). In particular, since this annihilator is a very well behaved object, we will be able to deduce that the central Oseledets bundle is continuous (actually smooth) in this class. On the other hand, we will show that this is not always the case by exhibiting a particular example where these object are genuinely distinct, and we will explain the main mechanism behind the zero exponents in this case.

Finally, if the time permits, we will follow a work in collaboration with A. Avila and J.-C. Yoccoz showing (among other things) that the central Oseledets bundle in this last particular example is not continuous (but only measurable).

Michal Misiurewicz *Random interval homeomorphisms*

We investigate two interval homeomorphisms, one moving points to the right and one to the left, applied randomly. We consider this system as a skew product with the Bernoulli shift in the base. Both homeomorphisms are piecewise linear and both endpoints are repelling in average. We prove that for almost all fibers the map is basically a contraction. We also prove the existence of a global (a.e.) pullback attractor, which is a graph of a function from the base to the fiber. However, the value of this function depends only on the past, so when we take the one-sided shift in the base, it disappears. This is a joint work with Lluis Alseda.

Sheldon Newhouse *Chaos for many parameters in the Lorenz system*

The Lorenz system is the three parameter vector field $X_{s,r,b}$ given by $X_{s,r,b}(x,y,z) = (s(y-x),(r-z)x-y,xy-bz)$. This has been studied extensively, mostly from a numerical point of view, as in the well-known book of Colin Sparrow, published in 1982. During the 1990's several researchers established the existence of homoclinic orbits and subshift dynamics for special parameters, using a combination of so-called 'shooting' methods and computer assisted proofs. We survey these results and present new results which show that the Lorenz system has topological entropy greater than $\log(2)/4$ for all parameters in a slight thickening the the parameter line segment

$$\{(s,r,b) : s = 10, b = \frac{8}{3}, 25 \leq r \leq 80\}.$$

The proof is computer assisted. It starts with a non-rigorous computational technique to identify "rectangles" with alleged mapping properties for the Poincare return map to the plane $z = r - 1$. It then uses a verified integration tool to make the mapping properties rigorous.

Anthony Quas *Semi-invertible multiplicative ergodic theorems and applications*

The Oseledets multiplicative ergodic theorem is of critical importance in many areas of mathematics, from smooth dynamics to probability theory. It has two formulations: an invertible formulation in which the underlying system and the matrices being multiplied are invertible; and a non-invertible formulation in these invertibility assumptions fail. We demonstrate that if the underlying system is invertible but the matrices are not, then many of the stronger conclusions of the MET still hold. This is of considerable significance for applications. We extend this to the case where matrices are replaced by linear operators. The principal motivation for this is the case of Perron-Frobenius operators.

Alexander Shnirelman *On the shooting problem for the Euler equations*

The flow of an ideal incompressible fluid in a bounded domain (or a compact Riemannian manifold) M can be regarded as a motion along geodesics on the group $D = \mathrm{SDiff}(M)$ of volume preserving diffeomorphisms of M equipped with the right-invariant L^2 metric. The 2-point (or shooting) problem consists of finding a geodesic on D connecting two given points ξ and η in D.

THEOREM. The shooting problem has a solution for any $\xi, \eta \in D$.

This theorem look superficially like the classical Hopf-Rinow theorem; however, these two theorems have little in common, and the proof is based on completely different ideas of "Microglobal Analysis".

Nandor Simanyi *Transversality of Singularities and Nonhyperbolicity Manifolds*

In this talk we will be studying the relationship between the singularities and nonhyperbolicity manifolds of hard ball systems in the 2-dimensional flat torus. The purpose is to sketch the main ideas of a new result that claims the transversality of the mentioned (codimension-one) manifolds. The proof presents a fascinasting interplay between techniques and tools of geometric and algebraic nature.

An important by-product of this transversality result is the long awaited completion of the proof of the Boltzmann-Sinai Ergodic Hypothesis on 2-tori.

Yakov Sinai *Binary Correlations of the Moebius function*

This talk is based on the continuation of our previous paper with F.Cellarosi and contains some estimates of correlations of the Moebius function.

Yingfei Yi *Noise Perturbation In Finite Dimension*

We consider white noise perturbations of a system of ordinary differential equations. By relaxing the notion of Lyapunov functions associated with the stationary Fokker-Planck equations, new existence and non-existence results of measure solutions in a general domain including the entire space will be presented for both non-degenerate and degenerate noises. Limiting behaviors of the measure solutions will be discussed along with applications to problems of measure stability and bifurcations.

James Yorke *The many facets of chaos*

Chaos is a concept with many facets or aspects. It has several definitions that emphasize different aspects of chaos. None is complete. My talk will illustrate how focusing on different aspects of chaos leads us in different directions and results in a fuller understanding of chaos. One aspect of chaos is the ability to control chaos. I will discuss the "partial control" of chaos.

Lai-Sang Young *Toward a smooth ergodic theory for infinite dimensional systems*

I will discuss some first steps toward building a nonuniform hyperbolic theory for infinite dimensional dynamical systems, focusing on settings that are consistent with those in systems defined by dissipative parabolic PDEs.

Vadim Zharnitsky *Hamiltonian dynamics in the linear search problem*

The linear search problem was introduced by Beck and Bellman in the early 60s. Assume an object is hidden on the real line according to a known probability distribution. The searcher wishes to find this object in the minimal expected time. What is the optimal search path? We study this problem from the view point of Hamiltonian dynamics. For the specific, yet representative case of exponential distribution, it is shown that the optimal path follows an unstable separatrix in the associated Hamiltonian system. This is joint work with Yuliy Baryshnikov.

Short talks.

Jianyu Chen (Penn State) *Coexistence of zero and nonzero Lyapunov exponents*

We consider examples of the coexistence phenomenon in smooth dynamical systems. To be precise, we can construct smooth systems which consist of two parts: one is regular with zero exponents, and the other is hyperbolic with nonzero exponents. Both parts are of positive volume. There are already several examples with such phenomenon in the category of volume-preserving diffeomorphisms. I shall present a volume-preserving flow of this type. We can also discuss the possibilities to find such examples in Hamiltonian systems.

Alexey Korepanov (Alabama) *Nonequilibrium stationary states in the thermostatted Lorentz gas*

I will talk about non-equilibrium steady states of a point particle in a two dimensional periodic Lorentz gas (Sinai Billiard). Particle is subject to a constant electric field as well as a Gaussian Thermostat which keeps the speed constant. Turns out that despite SRB measure is singular, its projections on the space coordinates are absolutely continuous and satisfy linear response laws.

Kiran Parkhe (Northwestern) *Distortion for diffeomorphisms of surfaces with boundary*

If G is a finitely generated group with generators $\{g_1, \ldots, g_s\}$, we say an infinite-order element $f \in G$ is a distortion element of G provided that $\liminf_{n\to\infty} \frac{|f^n|}{n} = 0$, where $|f^n|$ is the word length of f^n with respect to the given generators. Let S be a compact orientable surface, possibly with boundary, and let $\mathrm{Diff}(S)_0$ denote the identity component of the group of C^1 diffeomorphisms of S. Our main result is that if S has genus at least two, and f is a distortion element in some finitely generated subgroup of $\mathrm{Diff}(S)_0$, then $\mathrm{supp}(\mu) \subseteq \mathrm{Fix}(f)$ for every f-invariant Borel probability measure μ. Under a small additional hypothesis the same holds in lower genus. This generalizes a result of Franks and Handel to surfaces with boundary.

Ilya Vinogradov (Princeton) *Effective bisector estimate for $PSL(2,\mathbb{C})$ with applications to circle packings*

Let Gamma be a non-elementary discrete geometrically finite subgroup of $PSL(2,\mathbb{C})$. Under the assumption that the critical exponent of Gamma is greater than 1 we prove an effective bisector counting theorem for Gamma. We then apply this Theorem to the Apollonian circle packing problem to get power savings. The proof relies on spectral theory of $\Gamma\backslash PSL(2,\mathbb{C})$.

Ben Webb (Brigham Young) *Structural Transformations and Improved Stability Estimates of Dynamcial Networks*

A major obstacle in understanding the dynamics of a dynamical network (or a high dimensional dynamical system) is that the information needed to do so is spread throughout the various system components. Because of this it is tempting to find ways of concentrating this information while preserving some fundamental property or features of the system Rs dynamics. With this in mind we introduce the concept of an ISOSPECTRAL NETWORK EXPANSION. The idea behind such expansions is that a network's structure can be modified in a number of ways that preserve the system's dynamics while simultaneously concentrating the network's structural/dynamic information. This method allows us to give improved estimates for determining whether a dynamical network, or more generally a discrete-time dynamical system, has a unique global attractor.