

AP THEORY V: THERMODYNAMICS IN TOPOLOGICAL DISGUISE, GRAVITY FROM HOLOGRAPHY AND ENTROPIC FORCE AS DYNAMIC DARK ENERGY

H. E. WINKELNKEMPER

ABSTRACT. We exhibit a mathematically rigorous sub-theory of Artin Presentation Theory with many undeniable *qualitative* topological analogues to the paradigms of T. Jacobson, T. Padmanabhan and E. Verlinde, relating Gravity with Thermodynamics. We also point out some crucial differences; the latter have to exist, since AP Theory is a new, intrinsic, (i.e., model-free), theory, whose fundamental equation, the Artin Equation, is a non-analytic, purely discrete group-theoretic equation, thus avoiding, *ab initio*, nomographical and UV restrictions, which bother model-dependent analytic ODE/PDE using theories. Nevertheless, AP Theory strongly supports the new paradigms of the above theories in a clear *topological*, qualitative, (i.e., non-quantitative), mathematically rigorous manner, by revealing itself as a rigorous, 4D smooth topological Holographic Dynamic Dark Energy Theory. Its sheer meta-mathematical, very conceptually simple, mainly discrete group-theoretic existence, is the rigorous universal mathematical logic, justifying and enhancing Verlinde, et al.'s main non-technical qualitative arguments, especially as far as *Holography* and '*Entropic Force*' is concerned. Another short way of describing the theory is: we extend Bekenstein's celebrated holographic idea from differential *geometry* to 2D and 4D smooth *topology*, its ultimate, mathematically rigorous, background-independent, intrinsic, completion.

1. INTRODUCTION

Artin presentation theory is a very conceptually simple, new *holographic* infusion of discrete group theory, (in fact, pure braid theory), into the physically relevant theory of *smooth*, compact, connected, *simply-connected* 4-manifolds, with a *connected* boundary. Thus, to just understand the basics, it is sufficient to have the following few pages at hand, [W],[W2], section 2, and parts of the introductions of [W1], [C], [CW], as well as the Appendix in [RW]. All actual mathematical facts, used in this paper, are proven rigorously in these last four refereed published papers.

In this rather long introduction, (which is mostly about ultimate background-independent, topological holography, the main concept of this paper), we will take care to give exact page numbers, sometimes several pointing to the same facts, when describing the analogies of this paper with Verlinde's, and others, so that there is no need, at first, to read all of the references in full.

It is also very useful, from the beginning, to stress what we mean by 'intrinsic', 'model-independence', 'model-free', see [W2], p.2, [W3], p.3; it means that in AP Theory, we operate simultaneously on *all* 4-manifolds of the theory, from *the exterior*, as the AP Torelli group does, see ahead, [W], and p.3 of [W3]. We do *not* pick a given particular manifold and then study it *internally*, with analytic

PDE-using connections, their curvature and holonomy, foliations, etc., of its own particular differential geometry, as in AdS/CFT and other theories, e.g., [J], [J1], [P], [P1], [H], [EFS], et al. This augments the so-called background-independence of AP Theory. As the example of Cobordism Theory, (the intrinsic exterior version of the particular internal bordism and homology theory of a given manifold) shows, 'exterior' theories are crucial for solving 'internal' problems on manifolds, [W2], p.2.

Our general initial view is: AP Theory is the most *holographic*, background-independent, *intrinsic*, model-free, *4D smooth topological* analogue of Thermodynamics á la Bekenstein-Hawking, namely that their holographic '*black hole laws are just thermodynamics in geometrical disguise*'. The fact that AP Theory, except for the adjoining of the *2D* and *4D* smooth topological constructions of the next paragraph, is a purely discrete group-theoretic theory, makes it a very general, mostly *discrete* group-theoretic theory in the sense of the classical geometric Erlanger Program, [W4], and this represents its abstract, as holographic as possible, universal, *microscopic* 'thermodynamical' starting point. This is, so to speak, intrinsic, holographic *4D smooth topological*, (instead of differential-geometric), '*thermodynamics in 4D smooth topological disguise*'.

The basic topological construction in AP Theory of the smooth, compact 4-manifolds $W^4(r)$ from a discrete Artin presentation r of the fundamental group of their connected boundary, ([RW], p.631, [W1], p.239, p.250, [W2], p.2, p.4), i. e., *the most general mathematically rigorous version possible of holography in dimension 4D*, is our clear, explicit, mathematically rigorous, model-independent analogue of Bekenstein's celebrated heuristic holographic idea: that putting all information into the boundary will make the bulk collapse onto itself, see [B], creating a 'black hole'. Compare also to the heuristic 'causal set' approach, [Ho], p.8.

In AP Theory, though, we thus obtain a rigorously mathematically defined and physically relevant, compact, smooth, connected, simply-connected, 4-manifold, with a connected boundary, instead of a 'black-hole'.

This construction still makes sense, when the above fundamental group is the trivial group, (i.e., when, due to the now proven *3D Poincaré Conjecture*, we can assume the 4-manifolds are *closed* manifolds), and these are then the smooth *4D* topological, but mathematically rigorous analogues in AP Theory to 'black holes', as explained below.

This, (together with the cone-like, ∞ -generated at each stage, graded AP Torelli group, see below) is also our analogue to Verlinde's basic starting point, that '*space has one emergent holographic direction*', [V], p.10, p.3, p.27.

In fact, a discrete group-theoretic, topology-less, i.e., 'microscopic', (but not point-like!), Artin presentation, r , determines, (i.e., allows the emergence of) a 'macroscopic' compact, *smooth*, connected, simply-connected 4-manifold, $W^4(r)$, with a connected boundary, [W1], p.250, [W2], p.4, thus already revealing 'thermodynamical' features, but instead of 'geometric' ones, á la Bekenstein-Hawking, *2D and 4D smooth topological* ones.

This smooth topological *4D* construction, requires first a *2D* one: the discrete Artin presentation r first determines a diffeomorphism $h(r) : \Omega_n \rightarrow \Omega_n$, (of the compact 2-disk with n holes, Ω_n), which restricts to the identity on the boundary of Ω_n , and which is unique up to boundary-preserving isotopies of Ω_n , see [W1], p.225. Since any such diffeomorphism isotopy class also determines a unique Artin

presentation, we can consider this to be topological '*quantum smearing*', a qualitative, *2D smooth topological* analogue to Planck's 'continuous to discrete' Law, as explained in [W3], p.2. In AP Theory 'trans-planckian' becomes *2D smooth topological* in a mathematically rigorous manner.

In AP Theory, *2D membranicity*, represented by the $h(r) : \Omega_n \rightarrow \Omega_n$, is in a one to one correspondence, with discrete group-theoretic *crystallinity*, represented by the r .

Furthermore, this correspondence is strengthened by the simultaneous natural action on both of the AP Torelli group.

The above construction can also be interpreted as a, characteristic of *3D*, canonical analogue to 'topological insulators', see [QZ], [Wi1], p.16.

These $h(r) : \Omega_n \rightarrow \Omega_n$ are our intrinsic, model-free '*dynamic topological quantum holographic screens*', and simultaneously, our intrinsic background-independent *2D 'dynamic event horizons*', the last intrinsic, background-independent *2D smooth topological* remnants of Bekenstein's holographic black-hole '*area*' arguments, see also remark 3 in section 3 below. Notice that these diffeomorphisms can be iterated and already have a dynamical theory with classical topological entropy of their own (compare to [V], p.12) with Artin-Mazur zeta-functions, namely Nielsen-Thurston theory.

Thus in AP Theory, 'holographic screens' and 'event horizons', are not some kind of 'thought constructs' or 'imaginary boundaries', but actual mathematically rigorous, background-independent, intrinsic, 2D smooth topological concepts.

Compare this, see [B], with the classical background-dependent, non-intrinsic versions of holography, e.g., á la 't Hooft, Susskind, Maldacena, et al., (*'in quantum gravity the amount of information needed to describe a volume in a metric space-time can be entirely encoded on its boundary'*), as well with those of [Bo], [KP], [LKL], [ZGZ], [GV].

Our universal, background-independent, intrinsic, *4D smooth topological* version of this is: *the amount of 'information' needed to define the smooth 4D universe $W^4(r)$ can be encoded group-theoretically in its connected boundary.*

Indeed in AP Theory, this 'information' is a *discrete purely group-theoretic one*, a model-free 'gauge-theoretic' one, namely a certain *presentation*, an Artin presentation, r , ([W], [W1], p.225, [W2], p.7), of the fundamental group of the connected boundary of $W^4(r)$. Compare to [B].

This is the 'quantum information' from where 'gravity', i.e., the smooth *4D universes $W^4(r)$* emerge. This is a clear, most general, universal, *background-independent* relation between *4D smooth topology*, (instead of differential geometry), and discrete group-theoretic holographic information; compare to [Bo], [LKL1].

The fact that an Artin presentation r , a discrete, but non-point-like concept, actually determines the *4D smooth manifold $W^4(r)$* and its boundary, sharpens holography to the maximum: from dimension four to dimension 0, i.e., to discreteness, in fact, group-theoretic discreteness, i.e., the purest form of symmetry.

In AP Theory, holography is so deep that although all probabilistic statistical arguments are absent, one still could use the term 'entropic', when considering the action of the cone-like, ∞ -generated at each stage, graded group of smooth topology-changing Torelli transitions/interactions group below, [W], [W3], p.5. Compare to [G], p.947.

In AP Theory, holography is so strong that, when combined with the cone-like, ∞ -generated at each stage, graded action of the Torelli group, it deserves to be called 'thermodynamic' holography.

It is also natural to conjecture that this causes a certain 'chameleon-ness' property in AP Theory, [R].

The $4D$ smooth structures on the $W^4(r)$, represent the last $4D$ smooth topological remnants, (where GR 'time' does not appear explicitly, but can still be considered to be encoded in the *smooth* 4-dimensionality), of differential-geometric Einsteinian gravity in AP Theory, still allowing us to say, GR, *emerges holographically* from the *discrete* group theory of Artin presentations, which we consider to be the 'microscopic', but not point-like, dynamic 'vacuum'.

The discrete Artin presentations, the r , the '*atoms of spacetime*', see [P2], are intimately related to the $4D$ smooth spacetimes, the $W^4(r)$, via the dynamic $2D$ smooth membranic holographic screens, the $h(r) : \Omega_n \rightarrow \Omega_n$. Compare to [P2], [G], p. 945.

AP holography, *ab initio*, has the thermodynamical local to global, i.e., microscopic to macroscopic, feature related to the last remnant, (remaining after the radical holography and model-independence of AP Theory), of Einsteinian gravity. The construction of the $4D$ *smooth* spacetimes $W^4(r)$ from the discrete Artin presentations r , the '*atoms of spacetime*', still allows us to say GR is related to holography. Compare to [ADH], p.925, and [Wi].

In AP Theory, holography goes from the $4D$ *smooth topological* manifold $W^4(r)$ to the *discrete*, i.e., zero-dimensional, but not point-like, Artin presentation r . It is a subtle, but important fact that, even if r presents the trivial group, and thus the boundary of $W^4(r)$ is the $3D$ sphere, i.e., when we can suppose $W^4(r)$ is a *closed*, smooth 4-manifold, *non-trivial holography is still present*, even if, a priori, it seems 'there is no place where to put it', see remark 2 in section 3 below.

This again reveals the universality, basic strength, purity, depth and 'completeness' of discrete group-theoretic AP holography. Its discreteness should again be a cause of 'chameleon-ness', [R].

We remark that the, e.g., celebrated AdS/CFT holography, see [B], and more general attempts, e.g., [Bo], are not 'complete', in this sense, see remark 2 in section 3 ahead.

In AP Theory, in a purely mathematically rigorous manner, the strongest, most radical and complete holography is intimately related to the last 4D smooth topological remnant of differential geometric Einsteinian gravity.

In AP Theory holography in a mathematical rigorous, intrinsic, model-free manner, solves all the difficulties pointed out by Bekenstein, [B], of other holographic theories. Furthermore, it confirms that the only rigorous topological $(3 + 1)$ -QFT left standing topologically, after ultimate, radical, complete holography, is AP Theory as a whole, as explained on p.5 of [W3]. Compare to [B].

AP Theory is the most '*general framework that describes how space emerges together with gravity*', compare to [V], p.3, thus in a sense topologically 'completing' GR, also.

The cause of the existence of this radical universal holography is: the basic equation of AP Theory, the Artin Equation, is *not* an analytic, smooth continuum/classical ∞ -dimensional Hilbert space/path integral using, equation; it is, at each stage, n , a purely discrete group theoretic equation in the free group F_n , [W],

[W1], [W2], p.7, which characterizes pure, framed (colored) braids on n strands. Thus, a priori, e.g., analytic time-dependent, ODE or PDE equations and their nomographical UV restrictions, do not appear in AP Theory.

Group theory in AP Theory is there, ab initio; it is a model-independent 'given' and does not need to be obtained with metrics, curvature, holonomy and the integration of analytic equations on a particular given manifold.

Thus AP Theory is also intrinsically, universally, a background-independent, holographic, characteristic of $4D$, 'gauge theory', which goes beyond the model-dependent, classical group theory obtained from differential-geometric connections on a given manifold. See also remark 7 in section 3 ahead.

Artin presentations represent the ultimate discrete, but non-point-like, microscopic reality in AP Theory, and the above holography for the $4D$ spacetime universes, the $W^4(r)$, represents '*thermodynamics in smooth topological disguise*'. This is what the above heuristic Bekenstein-Hawking *geometric* analogy with thermodynamics logically and *topologically* has to lead to ultimately, in background and model-independent quantum gravity theories, see remark 7 in section 3 ahead.

Although at first classical black-hole theory led to the important concept of holography, classical point-like black-hole singularities do not exist on the smooth $4D$ manifolds $W^4(r)$ of AP Theory, unless one wants to call their connected boundaries black-holes. The universal, radical, as sharp as possible holography of AP Theory now goes beyond classical black-hole point-like singularities on a single, given smooth 4-manifold, due to the fact that, although discrete, i.e., zero-dimensional, Artin presentations are not point-like. However, as mentioned above and in section 2 ahead, AP Theory still has $2D$ smooth topological, *intrinsic* analogues to 'event horizons', the $h(r) : \Omega_n \rightarrow \Omega_n$ above, the last, but entirely mathematically rigorous, $2D$ smooth topological remnants of Bekenstein-Hawking black-hole 'area' arguments on a given manifold; see also remark 3 in section 3 below.

In AP Theory, the closest 'point-like' concepts are Artin presentations r of the trivial group, where then we can consider $W^4(r)$ to be a *closed*, smooth 4-manifold, see remark 2 in section 3 below. These should be analogues of '*holographic neutron stars*', instead of black holes, compare to [BPV], [Vi].

The above on holography is the typical example, characteristic of this paper, of comparing analytic equation-using, model-using, classical ∞ -dimensional Hilbert space using, path-integral-using, still heuristic theories, with the more general, model-independent, $4D$ smooth topological, mostly discrete group-theoretic, mathematically rigorous AP Theory.

We stress that AP Theory is *not* some sort of a particular meta-mathematical 'model', as, e.g., AdS/CFT theory, Horava Gravity theory, [H], etc. It is a cone-like, ∞ -generated at each stage, graded, mainly discrete group-theoretic, characteristic of $4D$, background independent, model independent, absolutely deterministic, parameter-free, non-perturbative, as holographic as possible, mathematically rigorous theory, (compare, e.g., to [BS] and references [7] and [8] therein). A priori, no new axioms, category/topoi theory, nor lattice theory, nor high-dimensional moduli arguments, nor graph theory combinatorics are necessary; compare to [ADH], p.924, and see also remarks 3 and 7 in section 3 ahead.

In addition to holography, the cone-like, ∞ -generated at each stage, graded group of smooth topology-changing Torelli transitions/interactions on the universes

$W^4(r)$, ([W], [W2],p.8, [W3],p.5), is our other main qualitative analogue to Verlinde's theory, (see section 2 ahead), namely '*the Entropic Force*', represented in AP Theory by the cone-like graded Torelli group action, which represents Holographic Dynamic Dark Energy, ([W3], p.8), as well as a dynamic, cone-like, ∞ -generated at each stage, graded 'tempered' version of classical ∞ -dimensional SUSY in AP Theory, [W3], p.6.

In AP Theory the *rigid* ∞ -dimensionality of classical Hilbert space and classical SUSY is substituted by the *dynamic* ∞ -generation at each stage, of the cone-like, graded Torelli group of topology-changing transitions/interactions, see [W], [W3], p.5.

Thus classical SUSY is only 'broken' by this cone-like canonical grading, which seems therefore to be a rigorous, intrinsic, model-free, canonical topological analogue of '*gauge symmetry breaking*', i.e., of '*superconducting*', compare to [Wi2], [Wi1], p.16.

This 'universal action reservoir' seems to have been heuristically intuited in [L], see also [M2], p.4, [Ch], [ZGZ].

This 'action' is again holographic, dynamically so, since it acts naturally, simultaneously, in unison, in a 3-fold way, on the discrete r , the $2D$ $h(r) : \Omega_n \rightarrow \Omega_n$ and the $4D$ $W^4(r)$. Compare to [KP], [M2].

Thus there is no mystery about how the Torelli action as a 'Entropic Force' acts on the microscopic, quantum gravitational level, compare to [M1], p.2, p.10.

Since at the microscopic 'atoms of space time' level of the static, inelastic, discrete Artin presentations, there are no Boltzmann statistics arguments, [P2], all such arguments going beyond the classical purely mathematical topological entropy of the diffeomorphisms $h(r) : \Omega_n \rightarrow \Omega_n$, [V],p. 12, become holographic dynamic dark energy arguments, instead of classical statistics entropy arguments. [W3], p.8.

Nevertheless, we think, due to the radical AP holography, going from $4D$ to the discrete r , Verlinde's term 'entropic force' is natural, although 'holographic dynamic dark energy' is more accurate, see also [LKL]. Compare to the remarks in [G], p.943, p.947.

In AP Theory, Boltzmann's temperature arguments, [P2], become part of the very strong cone-like, ∞ -generated at each stage, graded Torelli group: when acting on the microscopic r it gives a 'heat bath' [V], p. 25 and, simultaneously, when acting on the macroscopic smooth $4D$ universes, $W^4(r)$, it gives smooth topology-changing, holographic dynamic dark energy.

In AP Theory, *holography* and '*entropic force*', immediately emerge from first principles, namely the discrete group theory given by the Artin presentations r , which characterize pure framed (colored) braids, [W2], p.7.

The AP *cone-like*, graded 'entropic force', now dynamic dark energy, seems to be the last remnant of the 'Big Bang' in AP Theory, with the cone-like grading as the last remnant of 'thermal time', (compare to [CR]) and perhaps, the also cone-like, radical holography of AP Theory, i.e. the construction of the smooth $4D$ universes, the $W^4(r)$, should be called '*the Big Rip of GR*', compare to [CKW].

AP Theory gives a mathematically rigorous global, intrinsic, $4D$ *smooth topological*, mainly discrete purely group-theoretic umbrella to the new qualitative, non-technical physical paradigms of Verlinde, et al., without, a priori, any quantitative, analytical statements, e.g., such as analytic Newton or Einstein equations,

Einstein-Hilbert functionals, path integrals, quantitative equipartition, nor equilibrium, aether, speed of light, Lorentz invariance, 'unitarity' and 'observer' arguments, etc., which, if obtained rigorously, in AP Theory probably would have to be proven to be invariant under the Torelli smooth topology-changing 'Entropic Force'. This problem, which if it were indeed solvable, seems a very difficult analytical one, due to the radical $4D$ smooth topological change caused by the Torelli transitions/interactions, [W], [W1], and the encoding of 'time' mentioned above.

All our undeniable analogues to Verlinde, et al.'s concepts and ideas, in this paper, are of a mathematically rigorous, *qualitative*, $2D$ and $4D$ smooth topological, non-analytic, non-quantitative nature. In particular, any objections, (e.g., [G], [K], [LP], [COT]), to Verlinde's theory, if just, have to be concentrated on his technical, quantitative, classical analytical equation using parts.

Instead of relations to Newton's and Einstein's Laws, as in the theories of Verlinde, et al., AP Theory is more related to Poincaré's theories on electro-magnetism and early quantum mechanics, see [W3], p.1, p.12, remark 1.

Our interpretation of the 'Entropic Force' leads to an interesting question, (see also remark 1 in section 3 below): what does it mean, as far as modern cosmology is concerned, that in AP Theory, the Torelli entropic force is so subtle and strong, that it can actually change ('rip off') the whole *smooth* structure, (i.e., the last $4D$ smooth topological remnant of Einstein gravity), of a $(3 + 1)$ -smooth universe, $W^4(r)$, but leave the underlying purely *topological* structure of $W^4(r)$ intact? See the example in [C] and compare to, e.g., [CKW].

The sheer meta-mathematical existence of AP Theory already, (with the Torelli transitions as 'cosmological phase transitions'), at least topologically, seems to answer the title question of [T], in the affirmative.

It is difficult to imagine how a classic analytic theory of Dynamic Dark Energy could be able to explain these smooth $4D$ topology-changing 'gravitational phase transitions', a new Quantum Gravity phenomenon, rigorously, compare to [W3], p.12.

With AP Theory it seems that ultimate group-theoretic holography trumps any so-called 'Correspondence Principle', at least topologically. It is natural to conjecture that AP Theory is a No-Go theory with respect to the desideratum to obtain a Quantum Gravity theory which simultaneously 'approximates' classical General Relativity and some classical Quantum Field Theory; compare to [G], p.947.

Thus, in particular, AP Theory, with its sheer rigorous meta-mathematical existence, clearly delineates the meta-mathematical chasm between the ∞ -generated dynamic $4D$ smooth topological *qualitative* and the ∞ -dimensional Hilbert space, path integral and analytic differential-geometric *quantitative* using methods in Modern Physics.

2. E. VERLINDE'S THEORY WITHIN AP THEORY

In this section we explicitly pinpoint, as we did above with holography, other main intrinsic *qualitative*, smooth topological, i.e., non-quantitative, natural undeniable analogies and similarities, as well as some additions and deviations from them, in AP Theory, between concepts, ideas and facts of Verlinde's paper [V], which we assume the reader has at hand.

Roughly put, we completely 'intrinsicify' and topologize the qualitative geometric parts of Verlinde's heuristic, still partially non-intrinsic, background-dependent

extensions and analogies of black hole theory, [V], p.6, p.7, thus confirming and completing his heuristic holographic approach, at least as far as mathematically rigorous $2D$ and $4D$ smooth topology is concerned.

The above construction of the smooth universe $W^4(r)$, from the microscopic, but not point-like, discrete Artin presentation r , the 'atoms of spacetime', [P2], is already one analogue of Verlinde's p.2 '*gravity can emerge from a microscopic description...*'. It is discrete group theory, which is the analogue of 'information', as in his '*the central notion needed to derive gravity is information*'. Thus in AP Theory, discrete group theory, as in [W2], section 2, is the 'information' related to holography and gravity, compare to [B], [LKL1]. The microscopic, discrete Artin presentation r , contains all the information needed for the construction of the $4D$ smooth macroscopic universe, the $W^4(r)$, illustrating holography in its sharpest, most basic form. Compare also to [ADH], p.925, [LKL], p.2, [LKL1] and [KP].

In AP Theory, the intrinsic analogues of his '*event horizons*', '*holographic screens*' and their '*unknown dynamics*', [V], p.3, p.6, p.24, are simply the $h(r) : \Omega_n \rightarrow \Omega_n$, ([W1], p. 225, [W3], p.2), our $2D$ flat, membranic D-brane ground states, [V], p.24, which are used to construct the smooth $4D$ universes, the $W^4(r)$, in a non-local, non-infinitesimal manner, ([W2], p. 4), i.e., there is non-locality in these still semi-microscopic states, [V1].

As pointed out above, in AP Theory, the intrinsic 'event horizons', 'holographic screens', i.e., the $h(r) : \Omega_n \rightarrow \Omega_n$, are not 'imaginary boundaries' nor 'thought constructs' created to represent information in physical systems, [V], p.7, p.26, but actual mathematically rigorous topological objects, with two unambiguously background-independent, intrinsic 'sides', given by in which direction one wants to iterate the diffeomorphism $h(r) : \Omega_n \rightarrow \Omega_n$. They already have their mathematically rigorous topological entropy, which now is the '*finite entropy of the matter configuration*' of [V], p.12. See also remark 3 in section 3 below.

At each stage, n , the Ω_n , the static membranic part of our $2D$ topological 'holographic screens', are invariant under the action of the Torelli transitions/interactions, i.e., our 'entropic force', whereas the $h(r) : \Omega_n \rightarrow \Omega_n$ and hence the $4D$ universes, the $W^4(r)$, are in general not. Thus we can also say, like Verlinde, but intrinsically, that our 'entropic force', now represented by the Torelli action, '*changes the information associated with the positions of material bodies*', [V], p.1, p.3, when '*moving away from the holographic screen*'.

Thus in AP Theory, we also have a mathematically rigorous analogue to Verlinde's 'entropic' version of gravity, besides the last $4D$ smooth topological remnant of Einsteinian gravity, discussed above, namely the $4D$ smooth structures of the universes $W^4(r)$.

In fact both versions of 'gravity' could be described as the intrinsic "thermodynamic limit of the statistical mechanics of the 'atoms of space-time'", but where the discrete, microscopic Artin presentations as 'crystallic' inelastic 'atoms of space-time', due to being non-point-like, *do not need any classical statistical mechanics anymore*. In fact, AP theory is a completely deterministic theory, where holographic dynamic dark energy substitutes for a macroscopic 'entropic force'. Compare also to [G], p.943, p.947.

The whole cone-like, ∞ -generated at each stage, graded group of $4D$ changing Torelli transition/interactions is the analogue in AP Theory of Verlinde's so-called

Newton's potential ([V], p.12), which now gives the analogue in AP Theory of 'gravity', but as dynamic dark energy.

This version of 'entropic force' as 'gravity' goes dynamically beyond Einstein gravity in AP Theory, (see the end of the introduction above and remark 1 in section 3 below). It is so strong to be actually able to juggle the $4D$ smooth structures, i.e., 'gravities', on a fixed topological $4D$ manifold, exhibiting a new Quantum Gravity phenomenon.

In AP Theory, the $4D$ smooth topology-changing action of the Torelli group represent Verlinde's '*action at a distance*' as well as that '*motion and forces are consequence of entropy differences*', [V1].

Going beyond [V], p.26, the qualitative Planckian correspondence (see [W3], p.2) between the $2D$ smooth continuous $h(r) : \Omega_n \rightarrow \Omega_n$, now considered as 'dynamic event horizons', and the microscopic, discrete, but not point-like, Artin presentations r , represents a genuine, mathematically rigorous, $2D$ smooth topological analogue to classical radiation á la Planck, instead of Hawking or Unruh 'radiation'. See also remark 4 in section 3 below.

This topological 'quantum smearing' analogue to Planck's Law now also answers the title question of [M1] in the affirmative, at least topologically.

The intrinsic analogue of '*particles being dragged across the event horizons and holographic screens*', [V], p.7, is simply intuiting the classical fixed and periodic point theory, dynamics and entropy of the $2D$ $h(r) : \Omega_n \rightarrow \Omega_n$, which, again holographically, but now from $2D$ to $4D$, relate 'dynamic event horizons' to macroscopic GR gravity, i.e., the smooth $4D$ $W^4(r)$.

Thus, see [V], p.3: AP Theory is '*the general framework that describes how space emerges together with gravity.*'. Furthermore, AP Theory '*reverses the logic that lead people from the laws of gravity to holography*', since, with the graded, cone-like, radical AP holography, we construct the smooth $4D$ universes $W^4(r)$, and the Torelli action, i.e., 'entropic force', not viceversa, as, e.g., in [LW].

In AP Theory, the existence of holography is a mathematical construction, a genuine mathematically rigorous *theorem*, [W1], p.250, [W2], p. 4, not, see [V], p.27, '*..also an hypothesis, of course, and may appear just as absurd as an action at a distance*'.

In AP Theory, an analogue to Verlinde's 'entropic force', is the cone-like, ∞ -generated at each stage, graded group of $4D$ smooth topology-changing transitions/interactions, the Torelli group, [W], [W2],p.8, [W3],p.5, p.6.

This group acts *in unison*, simultaneously, in a 3-fold way, on the microscopic discrete, i.e., $0D$ r , the $2D$ D-brane ground-states $h(r) : \Omega_n \rightarrow \Omega_n$ and the $4D$ smooth universes $W^4(r)$. Thus there are no problems concerning the 'entropic force' acting on the microscopic quantum gravity level, compare to [M1], p.2, p.10.

Since there can not be anything more universal and dynamically stronger than this holographic, $4D$ smooth topology-changing, (i.e., 'cosmological/gravitational phase' transitions), ∞ -generated at each stage, cone-like graded action, emerging in a mathematically rigorous manner, from the vacuum of the discrete, i.e., microscopic, but not point-like, Artin presentations, *it has to be the analogue of Holographic Dynamic Dark Energy in AP Theory*, see p.8 of [W3]. These powerful ∞ -generated symmetries, which can actually change the whole $4D$ smooth structure on a 4-manifold, but leave the underlying purely topological structure intact, (see remark 1 in section 3 below), have to constitute a universal, cone-like,

graded, discrete group-theoretic *dynamic* analogue of the much more rigid classical SUSY, [W3], p.6. Thus AP Theory, without discarding all of GR, augments and strengthens our analogue to Verlinde's 'entropic force', to the *mathematically rigorous maximum*.

This should correspond to Verlinde's 'eye opener' epiphany, see [V1].

Verlinde's intuition regarding 'polymers', [V], p.4, p.25, [V1], and Hooke's Elasticity Law, [V], p.27, topologically, seems to be nothing but, intuiting the fact that, in AP Theory, the 'elastic' *2D membranics*, holographic screens, the $h(r) : \Omega_n \rightarrow \Omega_n$, hold together the microscopic discrete Artin presentations r with the macroscopic *4D* smooth universes, the $W^4(r)$. This universal 'cosmic tension', which is stable under the Torelli action, already again is a 'dark energy' property in AP Theory. It also could be called a 'cosmic' Local-Global principle.

In the intrinsic, non-quantitative AP Theory, 'entropic force' and holographic dynamic dark energy are essentially the same mathematical concept, compare to [EFS], [LKL], [ZGZ].

In AP Theory, 'action at a distance', [V], p.3, is defined by the powerful holographic topology-changing Torelli action. See remark 1 in section 3 below.

One can say, 'entropic energy' travels from one hologram to another, in which the *4D* smooth topology of the $W^4(r)$ is altered by that passage of 'energy'.

Compare to the critical remarks on this on p.2 of [KG], p.11 of [Ch], and also observe that, unlike as in [LW], in AP Theory, as in [V], p. 3, it is holography which clearly leads to the 'entropic force', not viceversa, and that just on the qualitative level, there is no contradiction with 'inflation', compare to [LP].

Other analogies with [V], [V1], and AP Theory are: their characteristic 4-dimensionality, the non-existence of moduli and classical high-dimensional string theory, the relevance of AP Theory to AdS/CFT, [W2], p.2, [V], p.23, the role of string theory as a UV-free (3 + 1)-QFT, see [W3], pp.5,7, [V1], as well, as Verlinde on p.26, pinpointing a '*somewhat subtle and not yet fully understood aspect*': in AP Theory, as mentioned above, the role of the quantitative Planck constant is substituted *qualitatively* by 'quantum smearing', the equivalence up to isotopies, between the *discrete* Artin presentation r and the corresponding smooth *continuous* *2D* D-brane groundstate $h(r) : \Omega_n \rightarrow \Omega_n$, (see p.2 of [W3]), which allow the construction of the *4D* smooth $W^4(r)$, see [W1], p. 250, [W2], p.4.

AP Theory and [V] also agree on the non-existence of gravitons of classical SUSY, see [V1], (but see remark 2 in section 3 below), high-dimensional string theory, [W3], p.7, (but see remark 5, in section 3 below), reversibility of the 'entropic force', and on the independence of the 'entropic force' and gravity from any particular field theory specifics, see [V], p.4, p.6, [W2], p.2.

AP Theory is immune to any qualitative criticisms, and is the group-theoretic universal rigorous 'Logic' of the non-technical, non-quantitative, non-analytic part of Verlinde's theory, especially as far as *holography* and '*entropic force*' is concerned.

AP Theory, due to its conceptual discrete group-theoretic, mathematically simple model-free universality, the fact that it is not a concocted mathematical *model*, and all the analogies above, has to be physically relevant, at least theoretically so, without any experimental verification. Basic discrete group theory has to be physically relevant, due to its pure basic dynamic and symmetric nature, especially when related in a fundamental manner, as above, to exotic smooth *4D*-manifolds, and, e.g., Donaldson/Seiberg-Witten Theory, see [W], [CW], [W2], p.9.

We end this section with an intriguing question: given an Artin presentation r , on n generators, let $A(r)$ be its exponential sum matrix, (see [W], [W1], p.226, [W2], p.7). They always are symmetric, integer $n \times n$ matrices, (the last finite remnants of classical Hilbert space in AP Theory, compare to [Wi]), which determine the homology of the $W^4(r)$ and their connected 3D boundaries, and are invariant under the Torelli action. They have the following 'logarithmic' property: $A(r \cdot r') = A(r) + A(r')$, where $r \cdot r'$ is the natural multiplication of Artin presentations, as in [W1], p.227, [W]. Thus one could write " $A(r) = \log r$ " and the question is: is this somehow a non-trivial, intrinsic, deterministic, non-statistical remnant in AP Theory of Boltzmann's or Bekenstein-Hawking's formulas for entropy?

Although, somewhat ironically, the Torelli Artin presentations, which dynamically create the 'entropic force' above, are defined by the condition $A(r) = 0$, see [W].

3. CONJECTURES, REMARKS, QUESTIONS, PROBLEMS

1. Concerning the 'entropic force', what does it mean cosmology-wise that in AP Theory, see [C], it has the dynamic power to change the diffeomorphism class of the smooth structure, i.e., the whole GR 'gravity', of a smooth 4D universe $W^4(r)$, *without* changing the underlying purely topological structure of $W^4(r)$?

Is this some sort of 'cosmic superconducting' phenomenon? Compare to [R].

Does this strong dynamic property of the AP Torelli 'entropic force', the smooth 4D topology-change, the encoding of time in the dimension 4, etc., uncover mathematical shortcomings in other, less universal, model-dependent, recent attempts at explaining and criticizing Verlinde's paper, e.g., [Ch], [CKW], [EFS], [G], [K], [KG], [KP], [LC], [LP], [LW], [LKL], [MR], [N], [P], [S], [Z]?

Another subtlety concerning the AP 'entropic force', is its behaviour with respect to Knot and Linking theory in the boundary of the $W^4(r)$, see the example in [W], [W1], p.230, [W2], p.8.

Furthermore, see [W], [C1], given any link L in any Z-homology 3-sphere, Σ^3 , there exists an Artin presentation r , where $A(r)$ is unimodular, such that $W^4(r)$ has Σ^3 as its boundary, and L is a sub-link of the the natural link defined in Σ^3 by the boundaries of Ω_n in $h(r) : \Omega_n \rightarrow \Omega_n$; the latter has the inverse of the matrix $A(r)$, (which defines the quadratic form of $W^4(r)$), as its linking matrix, see [W], [W1], p.248. Thus any link in any Z-homology 3-sphere is intimately related to a smooth, compact, simply-connected '4-brane', namely $W^4(r)$, whose boundary it is, compare to [W1].

2. When the Artin presentation r presents the trivial group, one can consider $W^4(r)$ to be a *closed*, smooth, simply-connected 4-manifold. (Non-trivial holography is still present, since, as is well known, *presentations* of the trivial group can be, a priori, as complicated as that of any group), and thus holographic difficulties such as, e.g., those of the AdS/CFT correspondence of p.6, p.7, of [Wi], do not occur in AP Theory. For example, see [CW], all complex Kummer surfaces, the only compact, simply-connected, complex Calabi-Yau surfaces, can be represented in this manner.

Should these closed, simply-connected, smooth 4-manifolds be considered as 'emergent gravitons' mediating in AP Theory's emergent gravity?

They should be the AP Theory analogues of the '*holographic neutron stars*' of [BPV]?

Should the set of all different Artin presentations of the trivial group, representing the same closed, smooth 4-manifold (up to diffeomorphisms) be considered as some sort of neutron/pulsar star 'radiation'?

3. Due to the obvious fact that the discrete, purely group-theoretic Artin presentations are more 'pure' mathematically, than the *group and graph theory mixing* 'spin networks' of Loop Quantum Gravity, AP Theory's holography is much sharper, and our Erlanger Program analogies with AP Theory, see [W4], seem to answer the '*geometrogenesis*' questions of p.12 of [S], see also [Ho], p.9.

Instead of classically relating $2D$ entropy with $2D$ 'area', or 'quantum area' as, e.g., in LQG, (see [S], [MR]) in AP Theory, more topologically, $2D$ entropy is related to the dynamic $2D$ D-brane ground-states, our dynamic, smooth topological quantum holographic screens, the $h(r) : \Omega_n \rightarrow \Omega_n$, [W1], p. 225, [W2], p.4, [W3], p.2.

4. Does the qualitative Planckian correspondence in AP Theory, [W3], p.2, i.e., AP 'topological quantum smearing', explain Hagedorn Temperature, where the Artin presentations, r , and the 'hadrons' $h(r) : \Omega_n \rightarrow \Omega_n$, which define a topological analogue of *confinement*, (see [W2], p.11, [W3], p.11), 'melt into', 'radiate into' each other?

5. In AP Theory, at its most microscopic level, string theory, ([W3], p.7.) unlike as in [V], p.3, [V1], is *not* emergent; the Artin presentations r , as 'quantum strings', are of a purely discrete nature, with smooth topology only emerging after our $2D$ smooth topological analogue of Planck's 'continuous to discrete' Law, [W3], p.2.

6. We conjecture that AP Theory should also be relevant for certain path-integral using theories, e.g., for obtaining a more general model-free extended Horava gravity theory, [H], [J1], without any Einstein Aether arguments. Are the $2D$ D-brane ground-states $h(r) : \Omega_n \rightarrow \Omega_n$, when acted upon by the ∞ -generated at each stage, graded, topology-changing, Torelli transitions/interactions, rigorous analogues to Horava's 'breathing mode' gravity waves? See [J1]. These diffeomorphisms also have entropy, an analytic one, coming from classical dynamics, see remark 1 on p.15 of [W4]. Notice here that AP Theory does not really 'de-couple' time; GR time can be considered to be encoded in the $4D$ of the smooth $W^4(r)$, after the aether destroying 'Big Rip of GR', so to speak, represented by model-independence and the radical holography used in the construction of the smooth $4D$ $W^4(r)$, that is, by the sheer meta-mathematical existence of AP Theory. Here it is also relevant, as mentioned above, that the last remnant in AP Theory of the Big Bang and 'thermal time', [CR], is just its cone-like, graded 'tempered' SUSY, see p.6, of [W3], now complemented by the Big Rip of GR, the radical holographic construction of the smooth $4D$ $W^4(r)$.

7. Due to to the topological similarities to Quantum Theory of the basic 'quantum smearing' continuous to discrete Planckian relation, (see [W3], p.2), of the $2D$ smooth D-brane ground-states $h(r) : \Omega_n \rightarrow \Omega_n$ to the *discrete* Artin presentation r , and the fact that the former define the $4D$ smooth manifolds $W^4(r)$, representing the last $4D$ smooth topological remnant of Einsteinian gravity, with the property of remark 1 above, AP Theory represents a very abstract, but non-trivial, *mathematically rigorous*, model-independent Holographic Quantum Gravity Theory, (i.e., a '(3 + 1)-TOE', by the skin of its teeth, so to speak), without any serious problems of 'time', compare to [CR], or 'information loss', or 'decoherence', etc. Compare to [Ho], [BS].

It also, at least topologically, completes and solves, if considered as a Theory of Topological Phases (namely the $h(r) : \Omega_n \rightarrow \Omega_n$ as 'string-nets'), the problems mentioned on p.1 of [LeWe], by giving that theory a rigorous mathematical background-independent group-theoretic framework.

Here 'quantum' and 'gravity' are held mathematically rigorously together by the sharpest, most far reaching holography: the construction of the $4D$ smooth universes $W^4(r)$ from the discrete, but not point-like, Artin presentations r , the atoms of spacetime and the 3-fold 'in unison' action of the cone-like ∞ -generated at each stage, graded Torelli group, on the r , the $h(r) : \Omega_n \rightarrow \Omega_n$ and the $W^4(r)$. Compare to the different, still heuristic, approaches to Quantum Gravity outlined in [Ho].

A moment of reflection shows that this is only possible, due to the non-analytical, non-differential-geometric, purely discrete group-theoretic Artin Equation defining the r , [W], [W1], p.225, [W2], p.7, but which nevertheless determine the $2D$ smooth flat 'membranes' $h(r) : \Omega_n \rightarrow \Omega_n$, in a *non-infinitesimal* manner, [W1], p.226, [W2], p.4, and these, then again in a non-infinitesimal manner, (by the smooth topological construction of p.250 of [W1], p.4 of [W2]), determine the smooth $4D$ universes, the $W^4(r)$, up to diffeomorphisms.

REFERENCES

- [ADH] Atiyah, M., Dijkgraaf, R., Hitchin, N., *Geometry and Physics*. Phil. Trans. R. Soc. A 2010 368, 913-926.
- [B] Bekenstein, J., *Information in the Holographic Universe*, Sc. Am. August 2003.
- [BPV] Boer, J., Papadodimas, K., Verlinde, E., *Holographic Neutron Stars*, arXiv:0907.2695v3 [hep-th] 23 Jul 2009.
- [Bo] Bousso, R., *The holographic principle*, arXiv:hep-th/0203101v2 29 Jun 2002.
- [BS] Bousso, R., Susskind, L., *The Multiverse Interpretation of Quantum Mechanics*, arXiv:1105.3796v1 [hep-th] 19 May 2011.
- [C] Calcut, J. S., *Torelli Actions and Smooth Structures on 4-manifolds*, J. Knot Theory Ramifications 17, 2008, 171-190. Available at <http://www.oberlin.edu/faculty/jcalcut/>.
- [C1] Calcut, J. S., *Knot Theory and the Casson Invariant in Artin Presentation Theory*, Fundamental and Applied Mathematics 11, L. V. Keldysh Memorial Proceedings, Moscow, 2005, 119-126. Russian Translation. Available at <http://www.oberlin.edu/faculty/jcalcut/>.
- [CR] Connes, A., Rovelli, C., *Von Neumann Algebra Automorphisms and Time-Thermodynamics Relation in General Covariant Quantum Theories*, arXiv:gr-qc/9406019v1 14 Jun 1994.
- [CW] Calcut, J.S., Winkelkemper, H.E., *Artin Presentations of Complex Surfaces*, Bol.Soc.Mat. Mexicana 10, 2004, 63-87. Special issue in honor of F. González-Acuña. Available at <http://www.oberlin.edu/faculty/jcalcut/>.
- [CKW] Caldwell, R., Kamionkowski, M., Weinberg, N., *Phantom Energy and Cosmic Doomsday*, arXiv:astro-ph/0302506v1 25 Feb 2003.
- [COT] Chaichian, M., Oksanen, M., Toreanu, A., *On Gravity as an entropic force*, arXiv:1104.4650v1 [hep-th] 24 Apr 2011.
- [Ch] Chivukula, A., *Gravity as an Entropic Phenomenon*, arXiv:1011.4106v2 [hep-th] 19 Nov 2010.
- [EFS] Easson, D., Frampton, P., Smoot, G., *Entropic Accelerating Universe*, arXiv:1002.4278v2 [hep-th] 24 Oct 2010.
- [G] Gao, S., *Is Gravity an Entropic Force?*, *Entropy* 2011, 13, 936-948.

- [GV] Garriga, J., Vilenkin, A., *Holographic multiverse and conformal invariance*, arXiv:0905.1509v3 [hep-th] 15 Oct 2009.
- [H] Horava, P., *Quantum Gravity at a Lifschitz Point*, arXiv:0901.3775v2 [hep-th] 2 Mar 2009.
- [Ho] Hossenfelder, S., *Experimental Search for Quantum Gravity*, arXiv:1010.3420v1 [gr-qc] 17 Oct 2010.
- [J] Jacobson, T., *Thermodynamics of space-time: The Einstein equations of state*, Phys. Rev. Lett. 75, 1260 (1995), arXiv:gr-qc/9504004v2 6 Jun 1995.
- [J1] Jacobson, T., *Extended Horava gravity and Einstein-aether theory*, arXiv:1001.4823v3 [hep-th] 11 Nov 2010.
- [K] Kobakhidze, A., *Gravity is not an entropic force*, arXiv:1009.5414v2 [hep-th] 30 Dec 2010.
- [KG] Kowalski-Glikman, J., *A note on gravity, entropy, and BF topological field theory*, arXiv:1002.1035v1 [hep-th] 4 Feb 2010.
- [KP] Kolekar, S., Padmanabhan, T., *Holography in Action*, arXiv:1005.0619v1 [gr-qc] 4 May 2010.
- [L] Lisi, A.G., *Quantum Mechanics from a Universal Action Reservoir*, arXiv:physics/0605068v1 [physics.pop-ph] 8 May 2006.
- [LC] Li, X., Chang, Z., *Debye entropic force and modified Newtonian dynamics*, arXiv:1005.1169v2 [hep-th] 20 May 2010.
- [LKL] Lee, J.-W., Kim, H.-C., Lee, J., *Gravity as Quantum Entanglement Force*, arXiv:1002.4568v1 [hep-th] 24 Feb 2010.
- [LKL1] Lee, J.-W., Kim, H.-C., Lee, J., *Gravity from Quantum Information*, arXiv:1001.5445v2 [hep-th] 21 Mar 2010.
- [LP] Li, M., Pang, Y., *A No-go Theorem Prohibiting Inflation in the Entropic Force Scenario*, arXiv:1004.0877v3 [hep-th] 8 Jul 2010.
- [LW] Li, M., Wang, Y., *Quantum UV/IR Relations and Holographic Dark Energy from Entropic Force*, arXiv:1001.4466v4 [hep-th] 30 Mar 2010.
- [LeWe] Levin, M. A., Wen, X.-G., *String-net condensation: A physical mechanism for topological phases*, arXiv:cond-mat/0404617v2 [cond-mat.str-el] 27 Apr 2004.
- [MR] Modesto, L., Randonò, A., *Entropic Corrections to Newton's Law*, arXiv:1003.1998v1 [hep-th] 9 Mar 2010.
- [M1] Munkhammar, J., *Is Holographic Entropy and Gravity the result of Quantum Mechanics?*, arXiv:1003.1262v3 [hep-th] 9 Mar 2010.
- [M2] Munkhammar, J., *Action in the Entropic Revolution in Newtonian gravity*, arXiv:1006.0723v1 [hep-th] 17 May 2010.
- [N] Nicolini, P., *Entropic Force, noncommutative gravity and ungravity*, arXiv:1005.2996v3 [gr-qc] 13 Aug 2010.
- [P] Padmanabhan, T., *Thermodynamical Aspects of Gravity: New insights*, Review Article. arXiv:0911.5004v2 [gr-qc] 19 Jan 2010.
- [P1] Padmanabhan, T., *Gravity: A New Holographic Perspective*, arXiv:gr-qc/0606061v2.
- [P2] Padmanabhan, T., *Grappling with Gravity*, Sc. Am. India, January 2011.
- [QZ] Qi, X., Zhang, S.-C., *Topological Insulators and Superconductors*, arXiv:1008.2026v1 [cond-mat.mes-hall] 12 Aug 2010.
- [R] Reich, E. S., *Dark Energy: is it the work of a cosmic chameleon?*, New Scientist, 31 June 2010.
- [RW] Ranicki, A., *High-Dimensional Knot Theory*, Springer, 1998. With an Appendix by H. E. Winkelnkemper. Available at <http://www.math.umd.edu/~hew/>.
- [S] Smolin, L., *Newtonian Gravity in Loop Quantum Gravity*, arXiv:1001.3668 [gr-qc] 20 Jan 2010.
- [T] Tkachev, I.I., *Gravitational phase transition: An origin of the large-scale structure in the universe?* Physical Review D, 45, no. 12, 15 June 1992.

- [V] Verlinde, E., *On the Origin of Gravity and the Laws of Newton*, arXiv:1001.0785v1 [hep-th] 6 Jan 2010.
- [V1] Verlinde, E., *Notes on [V]*, preprint, January, 2011.
- [Vi] Visser, M., *Black holes in general relativity*, arXiv:0901.4365v3 [gr-qc].
- [W] Winkelnkemper, H.E., *What is...an Artin Presentation?*, preprint 2003, 3pp. Available at <http://www.math.umd.edu/~hew/>.
- [W1] Winkelnkemper, H.E., *Artin Presentations I: Gauge Theory, (3+1)-TQFTs and the Braid Groups*, J.Knot Theory Ramifications 11, 2002, 223-275. Available at <http://www.math.umd.edu/~hew/>.
- [W2] Winkelnkemper, H.E., *AP Theory II: Intrinsic 4D Quantum YM Theory with Mass Gap*, arXiv:0711.2054v2 [math.GM] 22 Feb 2008.
- [W3] Winkelnkemper, H.E., *AP Theory III: Cone-like Graded SUSY, Dynamic Dark Energy and the YM Millennium Problem*, arXiv:1003.5147v1 [math.GM] 26 Mar 2010.
- [W4] Winkelnkemper, H.E., *AP Theory IV: Intrinsic Topological Quantum Langlands Theory*, arXiv:1010.5150v1 [math.GM] 25 Oct 2010.
- [Wi] Witten, E., *Quantum Gravity in De Sitter Space*, arXiv:hep-th/0106109v1 13 Jun 2001.
- [Wi1] Witten, E., *Five-branes and Knots*, arXiv:1101.3216 [hep-th] 17 Jan 2011.
- [Wi2] Witten, E., *From Superconductors and Four-manifolds to Weak Interactions*, BAMS 44, 2007, 361-391.
- [Z] Zhao, L., *Hidden symmetries for thermodynamics and emergence of relativity*, arXiv:1002.0488v4 [hep-th] 16 Apr 2010.
- [ZGZ] Zhang, Y., Gong, Y., Zhu, Z., *Modified gravity emerging from thermodynamics and holographic principle*, arXiv:1001.4677v1 [hep-th].

DEPARTMENT OF MATHEMATICS
 UNIVERSITY OF MARYLAND
 COLLEGE PARK, MARYLAND 20742
E-mail address: hew@math.umd.edu