Abstract. We show how Artin Presentation Theory, due to the fact that it contains a braid-theoretic topological analogue to Planck’s Law, should be considered as the topological completion of classical boson-fermion and fractional anyon statistics using QM. In AP Theory, all smooth $2D$ and $(3 + 1) − D$ topology is just in function of that of this analogue to Planck’s Law and does not need to be extracted from elaborate differential-geometric theories and their moduli. This allows genuine topological analogues to some problems of modern physics to be meaningfully stated, unified and solved in a non-perturbative mathematically rigorous manner. We then list applications to two important problems of modern physics, Quantum Gravity and YM Existence and Mass-Gap. One basic conclusion is that the unifying conceptual quantum topological simplicity of AP Theory is universally necessary to obtain mathematically consistent topological solutions to these problems, just as Planck’s Law was necessary to circumvent the infinities appearing in the original UV problem. It will become clear that AP Theory is the most basic Topological Quantum M-Theory, where M stands for membrane. The sheer mathematical existence of AP Theory, together with the above topological Planck Law, also makes it into a topological ‘scalar Higgs Field’, (thus avoiding the necessity of a ‘big bang’ hypothesis) and giving also, in particular, a topological unification of analogues of the strong force and gravity. AP Theory also strongly suggests that ‘mass’ and ‘gravity’ should originate in the ‘quantum vacuum’ and that classical strings as intuitive naive continuous arcs and their background-dependent ‘worldsheets’, should be made as obsolete as the ‘planetary’ electron orbits Heisenberg definitely got rid of by pure thought.

1. Introduction

Il faut triompher par la pensée, pas par le calcul. H. Poincaré

AP Theory is a rigorous non-perturbative, topological interacting Quantum Field Theory, characteristic of dimension 4, which, unlike analytic theories, has no mathematical obstructions in studying the ‘early universe’ with ‘vacuum catastrophes’, e.g., the Big Bang, since it is not based on analytic ODE /PDE equations, but is a purely braid-theoretic theory.

As a QFT, AP Theory immediately addresses Schwarz’s ‘third basic principle’, about why particles, which ‘in practice, are smeared over a region of space due to quantum effects, but their description in the basic equations is as mathematical points’; see [Sch].

AP Theory is a very conceptually simple theory, based on the mathematical concept of a pure, framed (i.e., coloured) braid and a purely discrete group-theoretic equation, the Artin Equation, that characterizes them. See [W], [AP2], section 2.

It is not a particular mathematical or physical ‘model’, but a conceptually autonomous, intrinsic, background-independent, deterministic, non-perturbative,
parameter-free, as holographic as possible, mathematically rigorous topological theory, which by its sheer rigorous mathematical existence singles out the physically most important dimension 4.

By autonomous and intrinsic here, we mean that smooth 4D topology arises only through an analogue to Planck’s Law, not by topologizing classic analytic differential-geometric constructs, and/or their moduli. Compare, e.g., to [Z], p.5.

This Occam razor-like conceptual simplicity, allows for a much greater mathematical unifying universality, than any analytic differential-geometric theory, or ∞-dimensional Hilbert space using theory. See also the introduction of [AP6].

Nevertheless, AP Theory is so general that it constructs and describes all closed, orientable 3-manifolds, together with their knot and linking theories, as well as, e.g., all simply-connected, complex elliptic E(n) surfaces, (including E(2), the Kummer surface). Hence, in particular, a non-trivial group-theoretic analogue to Donaldson/Seiberg-Witten theory exists in AP Theory, ([W], [AP1], [AP2], section 2) and thus, despite its conceptual simplicity, it has to be relevant to more elaborate 4D mathematical and physical theories.

Concerning Physics, AP Theory first topologically exploits the fact that the laws of Quantum Mechanics, permit in two dimensions, the existence of ‘anyons’, particles with ‘fractional statistics’, which is neither Fermi nor Bose.

It starts off with a braid-theoretic topological analogue of Planck’s classical ‘continuous to discrete’ Law, (see [AP3], p.2, and/or section 2 ahead), and only because of this fact, (now a theorem), constructs a theory of compact, connected, simply-connected, smooth 4-manifolds, \( W_4(r) \), each with a connected boundary, thus giving the last smooth 4D topological remnants and vestiges of Einsteinian Gravity in AP Theory.

In AP Theory, General Relativity is simply represented by the ‘landscape’, the ‘multiverse’ of the smooth 4-manifolds \( W_4(r) \).

This is the minimal price GR has to pay in order to fit into a mathematically rigorous QFT-like scheme such as AP Theory. Compare to [We], pp. 172, 191, 203.

The topological analogue to Planck’s Law also reveals itself as a ‘Higgs Mechanism’, see section 4 below.

Thus in AP Theory, we substitute ‘anyons’ and ‘fractional statistics’ by this non-probabilistic Planck Law and thus eliminate, topologically, the probabilistic words ‘fractional statistics’, and substitute them by topological certainty, i.e., in this case, topological determinism, compare to [Ho]. Indeterminism in AP Theory exists, see [AP2], p.11, but in a very precise manner, intimately related to ‘confinement’.

The basic ‘master’ equation of AP Theory, the Artin Equation, for each positive integer, \( n \), which characterizes pure, framed, (i.e., coloured), braids with \( n \) strands, (see [W], [AP1], [AP2], p.7) is a purely discrete, group-theoretic equation in the free group, \( F_n \), and, a priori, is not related to infinitesimal analytic nor ∞-dimensional Hilbert space, nor statistical concepts.

The discrete, but not point-like, purely group-theoretic Artin presentations, \( r \), characterize and are in one-to-one correspondence with all pure framed (i.e., coloured) braids, the intrinsic ‘bundles of energy’, [We], p.172, of AP Theory.

The discrete, but not point-like, Artin Presentation, \( r \) solves the problem of other QFTs of why, as emphasized by Schwarz in [Sch], (despite, in practice, elementary
particles, due to quantum effects, are 'smeared over a region of space'), the QFTs, in question, still give their description as actual points of analytic equations.

It will become clear that analytic differential geometric equations, and ∞-D Hilbert using methods, doing precisely this, is the cause of, e.g., the difficulties of bringing classical GR and QM together under one rigorous mathematical roof, i.e., in solving the so-called Quantum Gravity problem. See section 3 ahead.

In AP Theory, there do not exist any 'infinities', caused by point-like particles, see [We], p. 216.

AP Theory should be considered to be the meta-mathematical topological limit, which includes, a priori, the so-called 'big bang', (i.e., without any mathematical difficulties concerning it), of still heuristic physical theories and we will give many analogies of this fact. In particular, AP theory, is 'non-thermal' (see [AP5]) and features such as 'superconductance', Higgs field, maximal energy, etc., are inherent in AP Theory ab initio. We can consider AP Theory to be what is left after temperature has been maximized or minimized; AP Theory is beyond 'Planck Energy'.

We can say, topologically speaking, the sheer metamathematical existence of AP Theory, represents the Big Bang, since it substitutes it mathematically. Heuristic big-bang arguments are not needed in AP Theory.

It is natural to consider the discrete Artin presentation \( r \) to be a particle or 'quantum string' and the \( h(r) : \Omega_n \to \Omega_n \) as waves or membranes. Then the above Planck Law analogue is also the clearest most basic particle/wave duality as well as string/membrane duality. AP Theory unifies these two very basic dualities. Here \( r \) represents simultaneously both closed and open strings. This again addresses what Schwarz emphasized above.

The interactions/transitions of AP Theory consist of a cone-like, graded by the positive integers, \( \infty \)-generated at each stage, group, which we call the Torelli group, \([W],[AP3]\]. These smooth topology-changing interactions/transitions are both very subtle, as Example 3 in \([W]\) shows, and very strong: they can change the smooth structure of a \( W^4(r) \), leaving the underlying topological structure invariant, see \([C],[AP2]\), p.9.

They act simultaneously, in unison, preserving the grading, on all three 'landscapes': the 'Discretuum', 'quantum vacuum', 'zero-point field', given by the discrete, (but not point-like), purely group-theoretic Artin presentations, \( r \), on their corresponding 2D membranes \( h(r) : \Omega_n \to \Omega_n \) as waves or membranes. Then the above Planck Law analogue is also the clearest most basic particle/wave duality as well as string/membrane duality. AP Theory unifies these two very basic dualities. Here \( r \) represents simultaneously both closed and open strings. This again addresses what Schwarz emphasized above.

Thus, despite there being no boson and fermions to be able to define classical SUSY, the more 'anyonic' AP Theory has even a more genuinely dynamic SUSY, namely that of the graded, \( \infty \)-generated at each stage, topology-changing Torelli action principle, see \([AP3]\), section 2.

These strong symmetries make AP Theory the topological analogue of the classical maximally supersymmetric \( N = 4 \) Yang-Mills Theory, as well as at least topologically into a genuine Quantum Gravity Theory and 'Super Gravity' theory.

With the above reductive topological analogue to Planck’s Law one can avoid, a priori, all analytic differential geometrical, as well as \( \infty \)-dimensional Hilbert space methods in AP Theory, (in particular, \( \infty \)-giving moduli), just as Planck’s Law avoided the infinities in the UV catastrophe, due to the Raleigh-Jeans Law. Thus in AP Theory, any UV-like or big bang-like problems, e.g., the vacuum catastrophe,
with its 120 orders of magnitude too large cc conundrum, are avoided \textit{ab initio},
without invoking any renormalization nor ‘anthropic’ methods. Schwarz’s ‘third
basic principle’ for QFTs above, is always adressed.

In AP Quantum Gravity Theory, we always have ‘ultraviolet finiteness’, compare
to [BCJ].

In fact, topologically, the ‘Big Bang’ is similarly not a ‘vacuum catastrophe’ in
AP Theory. Compare also to Turok, [T].

The sheer mathematically rigorous existence of AP Theory represents ‘inflation’
and ‘flatness’ of the universe.

It is the sharpest mathematically rigorous TQFT in the sense of Atiyah and
Witten, due to having its own topological braid-theoretic analogue to Planck’s
Law and restricting its smooth topology to what this AP Quantum Law (now a
theorem, [W]) gives. It is a sporadic (3+1)-TQFT characterizing (3+1) dimensions
as explained in [AP3].

Concerning the Higgs boson, as opposed to the Higgs field, on the quantitative,
numerical level, the only prediction AP Theory makes at this point, is that the Nu-
merology of the Higgs boson should have many similarities with that of Poincaré’s
homology 3-sphere and its fundamental group, \(I(120)\), the binary icosahedral group,
a subgroup of \(SU(2)\) of order 120. See the Conjecture in section 7 below.

The group \(SU(2)\) is, due to its relation with Donaldson’s theorem, the only Lie
group appearing, a priori, in AP Theory, see [AP1], p.240, [AP2], p.9.

2. Immediate Conceptual Implications of the AP Topological
   Analogue to Planck’s Law

AP theory begins with a one-to-one correspondence between framed (coloured)
pure braids, with \(n\) strands, represented uniquely by an Artin presentation, \(r\),
with \(n\) generators, (i.e., a purely discrete, group-theoretic concept) and isotopy
classes, keeping boundaries fixed, of smooth diffeomorphisms \(h(r) : \Omega_n \to \Omega_n\),
which restrict to the identity on the boundary, \(\partial \Omega_n\), of \(\Omega_n\), the compact 2-disk
with \(n\) holes. (see [AP1], p.225, [AP2], p.7, [AP3], p.2.)

This topological analogue to Planck’s ‘continuous to discrete’ Law, (now a theo-
rem, [W], [AP1], not a postulate), gives a ‘duality’ between the smooth 2D contin-
um using \(h(r)\) and the discrete, but not point-like, purely group-theoretic Artin
presentation \(r\), which form the Discretuum, i.e., the ‘quantum vacuum’, ‘zero-point
field’.

One should recall here that Heisenberg’s ‘Helgoland Epiphany’ was essentially
to substitute analytic equations with discrete matrices. Thus by ‘pure thought’
he discovered the non-existence of ‘planetary’ electron orbits. In AP Theory, we
substitute analytic equations with the purely algebraic group-theoretic Artin pre-
sentations, also a purely discrete concept. Heisenberg restricted himself to studying
only ‘observables’, obtainable from his matrices. AP Theory restricts its topology
only to that obtainable, via \(r\), from its topological Planck Law.

Furthermore, just as, in Heisenberg’s case, the corresponding analytic continuum
dynamics, i.e., the Schroedinger picture, turned up at an equivalent mathematical
level, so in AP Theory, the analytic continuum, appears on an equivalent level,
(but non-infinitesimally!) via the \(h(r) : \Omega_n \to \Omega_n\) and the gravitational \(W^4(r)\).

This strongly suggests that the naive intuitive concept of ‘strings as actual con-
tinuous arcs’ and their background-dependent, swept-out in space ‘worldsheets’,
should be made as obsolete as ‘planetary’ electron orbits. In the back-ground-

independent AP Theory, the discrete, but not point-like, Artin presentation \( r \) is the string and the corresponding \( h(r) : \Omega_n \to \Omega_n \) is its intrinsic background-independent worldsheet.

At each stage \( n \) the Torelli group is isomorphic to the commutator subgroup of the pure braid group on \( n \) strands, which is infinitely generated, when \( n \) is larger than 2. Hence ‘non-commutativity’ is also well represented in AP Theory, but dynamically.

Roughly put, one can say: AP Theory braid-theoretically topologizes Heisenberg’s Helgoland Epiphany, starting from its braid-theoretic topological analogue to Planck’s original Law. See also remark 5 in section 7 ahead.

In AP Theory, this is also a proof of de Broglie’s particle/wave duality: the \( r \) are the discrete, (but not point-like) particles and the \( h(r) : \Omega_n \to \Omega_n \) are the waves. In AP Theory, waves have at least topological quantum ‘reality’.

It is instructive to consider the non-infinitesimal appearance of the smooth continuum here, as coming from the property of ‘fractional’ as it appears in the definition of anyons, (see [AP6]), instead of the classical continuum appearing from the rational numbers à la Dedekind. Thus the basic radically holographic, non-infinitesimal 2D smoothness of AP Theory, (given by the 2D membranes \( h(r) : \Omega_n \to \Omega_n \)), substitutes, at least topologically, for classical infinitesimal-using differential-geometric methods.

It is our contention, as suggested by Schwarz in [Sch], that it is the latter, by treating particles as mathematical points, which is the main obstruction in bringing classical QM and GR together, i.e., in solving the so-called Quantum Gravity problem in a non-perturbative rigorous mathematical manner, as well as unifying classical theories in the ‘early universe’. AP Theory is the clearest, most reductive, basic, conceptually simple and conceptually autonomous theory, which does this at least topologically, in a mathematically rigorous manner, (see section 3 below).

In this section, we first list some topological analogues to concepts in modern physics, which already are immediately intimately related to this topological Planck Law, and difficult problems of modern physics, which therefore can be solved at least topologically.

First of all, if we consider the discrete, but not point-like \( r \) to be particles and 2D membranes as a new type of ‘mass’, then the AP Planck Law ‘gives mass’ to any particle via the \( h(r) : \Omega_n \to \Omega_n \).

This immediately makes the whole of AP Theory into a topological Higgs Mechanism, a non-thermal Higgs Field, associated not with spontaneous symmetry breaking, but with a AP Theory graded analogue to SUSY. See [AP3], section 3.

In AP Theory, the quantum vacuum, i.e., the set of discrete, (but not point-like), Artin presentations, \( r \), gives membranic mass, i.e., the 2D \( h(r) : \Omega_n \to \Omega_n \), as well as 4D smooth gravity, i.e., the \( W^4(r) \).

The following also are already intimately related to this AP Quantum Law, and its autonomous, ‘anyonic’ topology, given by the smooth 2D membranes \( h(r) : \Omega_n \to \Omega_n \), thus at least unifying and relating different important classical problems topologically.
i) **Ultimate** \((3 + 1)\)-Holography. In AP Theory the discrete, (i.e., zero-dimensional, but non-point-like), purely group-theoretic Artin presentation \(r\) determines the smooth 4D manifold \(W^4(r)\), via the 2D membranes \(h(r) : \Omega_n \rightarrow \Omega_n\), in a rigorous mathematical fashion. In AP Theory, 'holography' is as rigorous and strong as possible and does not need 'to be derived from information of black holes'. In other words, the 'quantum string' \(r\) determines both the 'bulk' \(W^4(r)\) and its boundary simultaneously, avoiding many of the main problems (see, e.g., [GG]) of the still heuristic, also holographic AdS/CFT theory, which hopes to rigorously establish 'a duality between the string theory in the bulk and a field theory in the boundary'. For more on the radicality of AP holography, which gives the theory its topological mathematical universality, see the introduction of [AP5]. Compare also to [Th].

ii) **A SUSY Action Principle.**

Instead of classical SUSY between boson and fermions, in the topological more 'anyonic' AP Theory, SUSY is represented by the cone-like, graded Torelli group, (see [AP3], sections 2 and 3).

In AP Theory, see [AP3], section 3, SUSY is not 'broken', only graded in a cone-like manner.

These strong, \(\infty\)-generated at each stage, smooth topology-changing, graded symmetries exist despite the fact that AP Theory is a non-local theory, in the sense that no infinitesimal analyticity is used.

Furthermore they act simultaneously, in unison, preserving the grading, *externally*, on the particle, string \(r\), the wave, membrane \(h(r) : \Omega_n \rightarrow \Omega_n\) and the gravitational 'bulk' \(W^4(r)\) and its boundary. Compare to [We], p.194, [G], p.21.

It can be considered to be the dynamic Superforce, when considering AP Theory as a topological unified field theory/grand unification theory.

They make the whole of AP Theory into a 'charged' Higgs Condensate.

The Torelli should also be relevant to 'cosmic separation of phases', compare to [Wi].

iii) **Non-perturbative 4D Quantum Gravity.** The Artin presentation \(r\), via the \(h(r) : \Omega_n \rightarrow \Omega_n\), determines a compact, connected, simply-connected, smooth, 4-manifold, \(W^4(r)\), with a connected boundary. This is accomplished by a non-local, but smooth, topological construction without any use of infinitesimal analyticity, [AP1]. The only reason that these \(W^4(r)\) exist, is because of the AP topological Planck Law, nevertheless the smooth 4-manifolds involved still have an interesting Donaldson/Seiberg-Witten Theory, [W].

Thus AP Theory is *ab initio* already a Quantum Gravity Theory. See section 3 ahead.

iv) **Maximally Supersymmetric \(N = 4\) YM Theory.**

AP Theory, due to the Torelli symmetries is the maximally symmetric group-theoretic \((3 + 1) - D\) topological field theory, see [AP3], section 3, and compare to [Di]. The strong, \(\infty\)-generated at each stage, graded Torelli group of ii) above, makes AP Theory as symmetric as possible, ([AP3], section 2). It represents the dynamic energy that 'charges' the group theory of AP Theory and makes it into a genuine purely topological Gauge Theory. The Torelli form its intrinsic, universal, topological 'Lagrangian'. Compare to [AP3], section 2.

It does not need to be obtained from 'twisting four-dimensional \(N = 4\) Yang-Mills Theory', see [Wi], p.4, and compare to [AP4].
In fact, at this point of conceptual clarity and simplicity, it is the topological AP version of the latter. This is the prototype of the power of metamathematical reductive unification of the topology of AP theory, due to its analogue of Planck’s Law, which in particular, also makes AP Theory into a Higgs Mechanism.

AP Theory is a universal, non-perturbative, topological ‘quantization of YM theory’; it gives a topologically precise definition of the most general quantum gauge theory in 4D, by its sheer conceptually simple existence, without needing, e.g., non-rigorous path integral methods and/or too restrictive axiomatic quantum field theories.

A first important question here is: Does the Clay Millenium YM and Mass Gap problem have a topological analogue in AP Theory or does the latter dissolve into a non-problem or very simple one? Is it related to AP Theory as a topological Higgs field, Higgs condensate and Higgs mechanism? See section 4 below.

v) **Background-independent, non-perturbative, parameter-free (3+1)–D String Theory.** It is natural to consider r to be a discrete ‘quantum string’ and h(r) : Ω_n → Ω_n, a ‘dynamic membrane’.

Then string/membrane duality is exactly the AP quantum principle, which furthermore, via the construction of the ‘gravitational’ 4D smooth W^4(r), immediately makes the discretuum of the quantum strings, the r, into a quantum gravity theory; compare to [G].

AP Theory IS the ‘Stringscape’, see [Ch], [We], chapter IX, for many analogies of classical String Theory with AP Theory.

In AP Theory both ‘closed’ and ‘open’ strings are conceptually unified by both being represented by an Artin presentation r.

**Hence, a priori, there is no need to introduce Conformal Symmetry, into AP string theory, compare to [We], pp. 217, 308.**

In particular, in the braid-theoretic AP string theory, there is no need to appeal to dimensions larger than 4 to ‘obtain consistency’.

In AP Theory, via the topological analogue to Planck’s Law, there is an unification of, a priori, different concepts (‘forces unifying at the Planck scale’ in classical string theory), and immediate relation to gravity via the W^4(r), (‘existence of the graviton’ in classical string theory).

AP Theory also supports String Theory in a rigorous mathematical topological manner, with its intimate relation to SUSY and Knot and Linking Theory.

In AP string theory, the set of all the smooth, 4D manifolds, the W^4(r), is the ‘vacua’, the ‘landscape’, the ‘multiverse’, the ‘anthropic landscape’.

Just as in Cobordism Theory, see [AP2], p.2, a whole set of manifolds appears naturally, which nevertheless has relevance for studying such manifolds individually.

The AP analogue of Planck’s Law in fact makes AP Theory into the purest, rigorous, braid-theoretic, background-independent, topological quantum M-Theory, where M stands for membrane, and the whole AP Theory with its cone-like, graded by the positive integers, ∞-generated at each stage dynamic Torelli action is its own ‘light-cone frame’. Compare to [P].
vi) **Strong Force and Confinement.** In AP Theory the phenomenon of quark confinement is intimately related to the fixed point theory of the $h(r) : \Omega_n \to \Omega_n$, and thus to the AP topological Planck Law, just as all the above. See [AP2], p.11, [AP3], pp.4, 6, 11. This reveals the $h(r) : \Omega_n \to \Omega_n$ as topological quantum 'hadrons' with respect to their fixed points as 'quarks'.

In AP Theory, existing 'beyond' temperature, the 'quark-gluon plasmas' are represented by the diffeomorphisms, the dynamic membranes $h(r) : \Omega_n \to \Omega_n$ and their fixed points. Compare to [AP2], p.11.

***Thus AP Theory contains a topological analogue of the strong force, which now is intimately related to gravity via the $W^4(r)$.***

vii) **Hierarchy Problem.** [We], p.205. If we consider the discrete, but not point like, Artin presentations $r$ as particles, e.g., as 'composite fermions', see [AP6], p.5, and/or 'preons', see [BT], and the corresponding smooth 4D $W^4(r)$ as related to gravity, there is indeed a, a priori, big conceptual difference: 'gravity' in AP Theory is the only 4D smooth topological concept, which is intimately related to the Knot and Linking theory in the boundary of a $W^4(r)$, [W].

The AP Planck Law explains the so-called Hierarchy problem in the sense that the purely discrete, i.e., zero-dimensional, (but not point-like), crystaltic Artin presentations $r$ are quite different from the membrane-using 4D smooth $W^4(r)$. See also [AP3], p.8.

viii) **Vacuum Catastrophe, Cosmological Constant Conundrum.** With its topological analogue to Planck’s Law, AP Theory bypasses these in a similar manner that Planck’s original Law solved the original Raleigh-Jeans ultraviolet catastrophe, since infinities of analytic theories are absent in the purely braid-theoretic AP Theory; here one does not insist in considering particles as point-like and setting up ODE/PDE equations, [Sch], [We], p.216.

Hence AP Theory, with its topological analogue to Planck’s Law, and by restricting all its topology only to that given by it, serves as a universal topological mathematical unifier of deep problems in modern physics.

In the next sections, we elaborate more on the important classical unsolved problems, referred to in the Abstract above, but always staying as close to the AP analogue to Planck’s Law as possible.

3. **4D Non-perturbative Canonical Quantum Gravity: The Topological AP Solution**

What does the AP topological Planck Law do to solve topological analogues of the important questions of classical Quantum Gravity? It is, of course, already the shortest topological solution to the problem of relating the discrete (i.e. quantic) $r$, with the (gravitational) smooth 4D manifolds $W^4(r)$.

In AP Theory, 4D Quantum Gravity emerges from a planar 2D background, (the $h(r) : \Omega_n \to \Omega_n$, which are determined, via the AP Planck Law, by the discrete, but not point-like, Artin presentations $r$) as the 'graviton' in String Theory, but in a rigorous smooth, (but not infinitesimal), topological manner.

For a first survey of ideas and problems, see Blau, [B].

For example, AP Theory justifies such statements that 'QG requires String Theory', [B], p.1, since AP Theory starts off with the discrete Artin presentations $r$ as 'quantum strings'. But it is not 'the extended nature of the string', ([B], p.24),
that is responsible for the good UV behavior of the theory, but the logical jump to
topology, via the above AP analogue of Planck’s Law.

AP Theory has a fundamental non-analytic, discrete, group-theoretic ‘master’
equation, the Artin Equation, see [W], [AP1], instead of fundamental analytical
equations such as the Schroedinger equation in QM and the Einstein equations in
GR.

*It is our contention that this avoidance of analyticity and \( \infty \)-dimensional Hilbert
space methods is mathematically necessary in bringing QM-like and GR-like theories
under one consistent mathematical roof, even just topologically.*

AP Theory also topologically completes holographic area ideas on thermodynamics
of Bekenstein and Hawking, see [AP5], which presumably can only be explained
by a quantum theory of gravity, see p.9 of [B]. Compare also to Strominger’s First
and Second Problems on p.2 and p.3 of [S]; in AP Theory, his definition of “sharp”
[S], p.2, is substituted by the AP topological Planck Law.

AP Theory at least gives a rigorous topological theory for the classical Planck
formula and classical Bekenstein-Hawking formula, [AP5], [B], p.10, and furthermore is relevant to the cosmological constant conundrum as explained in [AP3],
section 4.

Unlike in LQG or in Horava-Lipschitz Theory, the also background-independent
AP Theory, does not, a priori, attempt a non-perturbative quantization of classical
GR, [B], p.15; its only relation to Einstein-like gravity is the landscape, multiverse
of the smooth 4D manifolds, the \( W^4(r) \), the last remnants of Einstein’s differential-
geometric gravity. Thus AP Theory supports the QFT viewpoint of Camp S of
[B], p.16, see also [AP3].

AP Theory, via the microscopic \( r \) and the macroscopic \( W^4(r) \), *topologically is
the covariant quantization approach* to Quantum Gravity, compare to [H], p.4.

AP Theory is a rigorous topological, UV completion of both classical QM and
GR, compare, e.g., to [BCJ].

We stress that even just (consistent mathematical) topological theories, ‘do not
grow on trees’, [B], p.23, and AP Theory should be very relevant to more elabo-
rate mathematically consistent analytical theories, if indeed they exist, e.g., the cc
problem, Higgs Field problem and other Hierarchy problems.

This brings us to the next section, compare to [B], pp. 27, 28.

4. YM Existence and Mass-Gap: the Topological AP Solution

In this section, using the AP Planck Law, we make our qualitative topological
arguments of section 3 of [AP2] and section 5 of [AP3] more explicit and precise,
and point out some similarities to two other analytic metric-using, but also non-
perturbative, attacks on the YM Millennium problem by Dynin, [D], and Kholodenko,
[K]. A big difference with our approach though, is that it is universal, ‘exterior’,
‘global’ and does not restrict itself to working ‘inside’ a particular 4-manifold, see
[AP5], p.1.

Due to iv) in section 2 above, [AP2], section 3, and [AP3], sections 3 and 5, it is
clear that AP Theory is the topological analogue of the maximally supersymmetric
\( \mathcal{N} = 4 \) Yang-Mills Theory, thus, at least topologically solving the existence part of
This despite there being no analytical equations in AP Theory. The basic equation of AP Theory is the Artin Equation in the free group, see [AP2], p.7, which characterizes pure framed braids.

The crucial question, regarding the very elusive concept of mass, (already adressed in section 3 of [AP2], section 5 of [AP3]), is:

What is the sharpest qualitative smooth, topological analogue in AP Theory, considered now as a Topological Quantum Yang-Mills Theory, to the Mass Gap problem?

Quoting from The Conversation (http://theconversation.edu.au) 9 December 2011:

"The Millenium problem seeks to establish by rigorous mathematics the existence of the 'mass gap', that is, the non-existence of massless particles in Yang-Mills theory. The solution to the problem would involve an approach to QFT in 4 dimensions that is sophisticated enough to explain at least this feature of quantum non-abelian Yang-Mills gauge theory."

We claim braid-theoretically in AP Theory it is the following:

If we consider the discrete, (but not point-like), Artin presentation \( r \) as a mass-less particle and the membranicity of the corresponding \( h(r) : \Omega_n \to \Omega_n \) as 'mass' in AP Theory, then it is precisely the AP topological analogue of Planck’s Law, which 'gives mass to the particles \( r \)'.

The 'gap' is between the mass-less discrete \( r \) of the discretuum, i.e., the quantum vacuum, and the membranic, massive \( h(r) : \Omega_n \to \Omega_n \).

This is the Mass Gap in the non-perturbative topological anyonic setting of AP Theory.

It is the braid-theoretic qualitative analogue to the analytic quantitative mass gap of [JW], p.3. Compare also to p.10 of [AP3].

This is obviously the sharpest topological analogue to the mass gap one can state in AP Theory and the gap is 'bridged', so to speak, by the topological AP Planck Law.

In AP Theory, topologically, the solution to the Millenium YM Existence and Mass Gap problem reveals itself as a smooth 2D topological Higgs Mechanism as explained above, see also [AP3], p.9.

The whole of AP Theory IS the Higgs Field and Higgs Mechanism, due to the AP Planck Law.

AP Theory actually defines 'membranic mass' with its topological analogue to Planck’s Law.

This is another example of the unifying power of the autonomous topology of AP Theory.

No new axioms nor metrics are needed in AP Theory to define the mass gap; it suffices to recognize the AP Planck Law as a rigorous particle-wave duality, the discrete, (but not point-like),particle, \( r \), and the 2D \( h(r) : \Omega_n \to \Omega_n \) 'standing' membranic wave.

This is also in mathematical accord with String Theory: the dynamic membrane \( h(r) : \Omega_n \to \Omega_n \) is the smooth topological 'tension' of the discrete, 'crystallic', quantum string \( r \).

Thus the Standard Model of particle physics is motivated at least topologically, giving a hint of its relevance in cosmology.
If now we remember that the cone-like, graded by the positive integers, \( \infty \)-generated at each stage, graded group of topology-changing interactions/transition, the Torelli, (see section 2 of [AP3]), should be the Dynamic Energy in AP Theory, AP Theory should also itself, in its totality, be considered a universal, intrinsic, topological 'Energy-Mass Spectrum', compare to [D].

'Energy' is represented by the powerful \( \infty \)-generated at each stage, graded Torelli dynamics and 'mass' by the 2D membranic \( h(r) : \Omega_n \to \Omega_n \), with which the gravitational smooth 4D manifolds \( W^4(r) \) are constructed in a non-infinitesimal manner.

Here also it is relevant that the Torelli act simultaneously, in unison, 'superconductively' on the discrete, i.e. zero-dimensional, (but not point-like) \( r \), i.e., the discretuum, the membranic smooth 2D \( h(r) : \Omega_n \to \Omega_n \) and the gravitational smooth 4D \( W^4(r) \).

In particular, the Torelli action on the discretuum, i.e., the 'vacuum', that is 'vacuum energy', is closely linked the the mass gap as well as to holography, see [Th].

Thus AP Theory, considered as 4D topological quantum Yang-Mills theory, has a mass gap and a cone-like tower of excitations provided by the graded Torelli group.

Furthermore, as in [K], 4D gravity is immediately related, via the smooth 4D \( W^4(r) \), as well as to 'confinement', and Knot and Linking theory in 3-manifolds, as explained in the previous sections, and references therein.

As in [D], AP Theory suggests that, e.g., the Garding-Wightman axioms, although consistent, might be too strong to successfully attack the classical YM Millenium problem, in its original analytic form.

A pertinent question, (already adressed in [AP2], p.12, [AP3], p.12), here is: How relevant is this braid-theoretic 'cosmic' topological solution to the actual analytic Clay YM Existence and Mass Gap problem, as stated in [JW]?

Would a most general analytic solution, be doomed, due to the powerful smooth topology-changing Torelli, (see [AP3], section 2), thus logically showing that the non-locality, non-analyticity of AP Theory, in accordance with Schwarz's third basic principle, [Sch], is necessary for a mathematically rigorous general solution, like the above topological one? See [AP3], p.12.

AP Theory supports the arguments of Laughlin and Pines, [LP] against the existence of TOEs which use analytic equations, etc.

How far would AP Theory constrict the generality of a consistent classical analytic differential-geometric solution of the the Clay YM Existence and Mass Gap Problem, as stated in [JW]?

5. More on the Unifying Conceptually Autonomous Topology of AP Theory

Just as classical GR restricts itself with the bound on all velocities by that of light, and classical QM restricted itself with the Uncertainty Principle to predicting experiments statistically instead of studying the whole of 'nature', AP Theory restricts its topology only to that given by its analogue to Planck's Law, and that is the secret of its universal rigorous mathematical unifying power. Compare also to [Z], p.5.
Augmenting AP Theory’s ‘physicality’ is the existence of a very powerful \textit{dynamic} theory, due to the cone-like, $\infty$-generated at each stage, topology-changing Torelli transitions/interactions, see [AP3], section 2.

In AP Theory, waves have at least braid-theoretic topological quantum reality.

AP Theory, (a theory, \textit{not} a model), with its Planck Law, in particular, as a Higgs Mechanism, holographically unifies the topological analogues of many important basic concepts in Modern Physics, e.g., particle-wave duality is the same as string-membrane duality, the Clay YM Existence and Mass Gap problem is essentially the same in AP Theory as a membranic mass giving Higgs Mechanism, mass and gravity originate in the quantum vacuum, etc.

Furthermore, one could say, the 'Big Bang', (at which all analytical theories fail, due to their equations blowing up to $\infty$), is nothing but the sheer 'instantaneous' metamathematical existence of AP Theory as a rigorous, very conceptually simple topological $(3+1)$-QFT theory, with all the cone-like properties, graded by the positive integers, described above and in [AP3].

In AP Theory, the whole 'landscape', 'multiverse', is studied in unison; unlike as in [D], [K], it does not pick a particular 4-manifold and studies it internally, with its own differential geometry as explained in [AP5], pp.1,2. Compare also to Weinberg’s statement on the 'symmetries of equations', [We], p.194.

The Torelli interactions/transitions act instantaneously on the whole landscape, on the whole multiverse.

Here 'instantaneous' also should imply 'maximum/minimum' temperature, i.e., non-thermal, beyond temperature, 'superconducting', beyond Planck energy.

Thus, e.g., the string-theoretic 'branes colliding' approach to the so-called Big Bang Model by Turok, et al., [T], has more support by AP Theory, at least topologically, than any classical standard approach to the Big Bang.

Topology in AP Theory, a priori, does not arise by topologizing analytic differential-geometric constructs and/or their moduli, and thus, unlike analytic theories, is not disturbed nor obstructed mathematically, by any 'big bang' hypothesis.

The conicity of the grading of the powerful Torelli dynamics of AP Theory represents the so-called big bang.

In other words, AP Theory would 'still hold at the Big Bang', just as Planck's original theory still held at the UV catastrophe.

For example, in AP Theory, a 'quark/gluon plasma' at extremal temperature, is rigorously mathematized topologically by the concept of the $h(r) : \Omega_n \to \Omega_n$ above and their fixed and periodic points, [AP2], p.11.

In AP Theory, strong force, the quarks and hadrons, the $h(r)$ are united with gravity, the $W^4(r)$.

Analogues of the unification power of the classical Higgs Mechanism still hold in the case of the strong force, represented by the $h(r) : \Omega_n \to \Omega_n$, and gravity, represented by the $W^4(r)$, as described in the previous sections.

In [We], Chapter IX, the reader can find more genuine analogies of AP Theory with known physical theories.

Finally, we again ask: Is the sheer existence of AP Theory beneficial for solving the above important physical problems \textit{analytically} with conventional methods? Can AP Theory be topologically destilled from such hypothetical analytic theories? Or should the sheer meta-mathematical existence of AP Theory be actually an obstruction?
It is our contention that it is the latter, due to among other things the \( \infty \)-generated groups at each stage of the smooth topology-changing Torelli group. See also [AP3], p.12, [AP3], p.12.

In the next section we point out some qualitative, braid-theoretic, topological, (not analytical, quantitative) analogies of AP Theory with Witten’s article [Wi2].

6. Superconductance, 4-manifolds and Strong Interactions

Superconductivity in AP Theory is represented by the fact that it is a non-thermal Higgs field already, and the fact that the powerful Torelli act simultaneously, in unison, on the \( r \), the \( h(r) : \Omega_n \to \Omega_n \) and the \( W^4(r) \); AP Theory in its totality is already a charged Higgs gauge field as explained above.

This is, in AP Theory, the analogue of Witten’s analogy, ([Wi2], p.361), of Higgs-Englert-Brout ‘straight forward relativistic model of electroweak symmetry breaking and the Landau-Ginzburg model of superconductivity’.

In AP Theory ‘electricity’ is represented by the crystallic discretuum of the discrete, but not point-like, Artin presentations and ‘magnetism’ by the dynamic membranicity of the associated diffeomorphisms \( h(r) : \Omega_n \to \Omega_n \). The AP Quantum Law, a consequence of the Artin Equation, makes the latter do braid-theoretically what the Maxwell Equations do analytically with PDEs and connections. This seems to be the cause of the amazing quote of Poincaré in [AP3], if one remembers that he was an expert in early Quantum Theory, as well as the father of modern topology. Concerning this, see also [AP3], p.12.

The Torelli then represent intrinsic, braid-theoretic, non-abelian quantum topological gauge theory.

Thus the Meissner effect, (magnetism dissipating during superconductance), is reversed by the AP Planck Law, in accord with AP Theory being superconducting ab initio. In the superconducting medium of AP Theory, the AP Quantum Law unites Electricity and Magnetism, in AP Theory, not analytically, but with the simplest framed pure braid theory.

It should be useful to further see how AP Theory, which also includes strings, strong force and gravity, is a meta-mathematical braid-theoretic topological limit of Witten’s paper, [Wi2], see his last section 4.7.

7. Remarks, Questions, Conjectures

1. The AP Planck Law, considered as a Higgs Mechanism, actually says what mass in AP Theory actually is, not just what it does. Mass for the discrete, but not point-like, particle \( r \) is the membranicity, the non-infinitesimal, 2D flat continuum of \( \Omega_n \) in the corresponding \( h(r) : \Omega_n \to \Omega_n \).

2. Due to the truth of the Poincaré Conjecture, (Perelman’s Theorem), the \( W^4(r) \), when \( r \) presents the trivial group, can be considered to be closed, smooth 4-manifolds. They include all complex elliptic \( E(n) \) surfaces, in particular the Kummer surface, \([W]\). Are they the equivalent, in AP Theory, of Gravitons, Quark stars and/or Neutron stars? Compare to [AP5], p.2. Are they the equivalent in AP Theory of classical Black Holes, i.e., now ‘pockets of dark energy’ and/or ‘fuzzballs’? (Classical black holes on a given 4-manifold, a priori, do not exist in AP Theory). This supports the ideas of Chapline, Laughlin, et al.
AP Theory also supports the arguments against the big bang of Turok, [T], and those of Laughlin and Pines, [LP], against TOEs based on analytic and other types of equations.

3. **Conjecture**: The numerology for the LHC Higgs boson should be similar to that of the Poincaré homology 3-sphere and its fundamental group, $I(120)$, the binary icosahedral group, a discrete subgroup of $SU(2)$ of order 120.

Is this also relevant to answering the important question why in the ‘vacuum catastrophe’, the classical cosmological constant is 120 orders of magnitude larger than the observed one? See [AP3], section 4.

4. In AP Theory should the Higgs Boson be the photon of Dark Energy? See section 4 of [AP3].

5. In AP Theory, should the braid-theoretic Artin Equation, [W], be considered a correct braid-theoretic topological quantum Heisenbergian ‘Weltformel’ of some sort, since it rigorously has the Quantum Gravity, Higgs Field and other important properties above?

Here it is relevant that the Artin presentation $r$ defines a discrete group, namely the fundamental group of the boundary of $W^4(r), ([W], [AP1], [AP2], section 2). The quantum string $r$ encapsulates even more ‘symmetry’ in this manner. Does this represent ‘isospin’ in AP Theory?

In AP Theory, the strong $\infty$-generated global Torelli symmetries, (i.e., the analogue of classical SUSY in AP Theory), mix well with these more local symmetries; there are no ‘NO-GO theorems’ preventing that.

Should AP Theory support, at least braid-theoretically, the conjecture of Banks-Fischler-Shenker-Susskind on M-Theory? See [P].

6. Does AP Theory do to all standard analytic differential-geometric physical theories what Heisenberg did with the analytic ‘planetary’ electron orbits?

7. In 1898 Poincaré, see [ZT], proposed an approach to the uniformization problem for Riemann surfaces, based on Liouville’s equation on the compact $2D$ disk with $n$ punctures, which contains the set $\Omega_n$ above. Since this is connected with an approach of Polyakov to QFT and string theory, [ZT], p.143, there is a hope that the pure smooth topology of AP Theory, could be related to these analytical methods and the Teichmüller space of $\Omega_n$. Compare also to Poincaré’s ‘resonateurs’ in [AP3], p.12.

References


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