Math 241 Exam 3  Dr. Justin O. Wyss-Gallifent

Directions: Do not simplify or evaluate unless indicated. No calculators are permitted. Show all work as appropriate for the methods taught in this course. Partial credit will be given for any work, words or ideas which are relevant to the problem.

Please put problem 1 on answer sheet 1

1. (a) Write down the iterated double integral for \( \int \int_{R} x \, dA \) where \( R \) is inside \( x^2 + y^2 = 4 \) and above the \( x \)-axis treated as horizontally simple. Include a picture of \( R \). \[8 \text{ pts}\]

(b) Write down the iterated double integral in polar coordinates for \( \int \int_{R} y \, dA \) where \( R \) is the region inside \( x^2 + y^2 = 4 \) and to the left of \( y = x + 2 \). Include a picture of \( R \). \[12 \text{ pts}\]

DO NOT EVALUATE.

Please put problem 2 on answer sheet 2

2. (a) Draw the surface \( \vec{r}(x, z) = x \hat{i} + (4 - x^2) \hat{j} + z \hat{k} \) for \(-2 \leq x \leq 2 \) and \( 0 \leq z \leq 3 \). \[8 \text{ pts}\]

(b) Evaluate \( \int_{-2}^{2} \int_{0}^{2} ye^{(x^2)} \, dx \, dy \). \[12 \text{ pts}\]

Please put problem 3 on answer sheet 3

3. (a) Draw, label and shade the region \( R \) parametrized by \( \int_{0}^{\pi/3} \int_{1+\cos \theta}^{2} \, dr \, d\theta \). \[7 \text{ pts}\]

(b) Write down an iterated triple integral in rectangular coordinates for the volume of the solid \( D \) where \( D \) is bounded by the coordinate planes and the plane \( x + y + 3z = 6 \). \[13 \text{ pts}\]

DO NOT EVALUATE.

Please put problem 4 on answer sheet 4

4. (a) Write down the iterated triple integral in cylindrical coordinates for the volume of \( D \), the solid inside \( x^2 + y^2 + z^2 = 9 \) and above the plane \( z = \sqrt{5} \). Draw pictures of \( D \) and \( R \). \[10 \text{ pts}\]

(b) Let \( D \) be the solid inside the cone \( z = \sqrt{3x^2 + 3y^2} \), above the sphere \( x^2 + y^2 + z^2 = 1 \) and below the plane \( z = 3 \). Suppose the mass density of \( D \) at \((x, y, z)\) is equal to \( z \). Write down the iterated triple integral in spherical coordinates for the mass of \( D \). \[10 \text{ pts}\]

DO NOT EVALUATE.

Please put problem 5 on answer sheet 5

5. Perform a change of variables which changes the integral \( \int \int_{R} x \, dA \) to an iterated double integral over a rectangle in the \( uv \)-plane. Here \( R \) is the region bounded by the lines \( y - x = 0 \), \( y - x = 2 \), \( 3x + y = 0 \) and \( 3x + y = 4 \). Make sure that all your steps are clear and draw both your regions \( R \) and \( S \). Your final answer should be an iterated integral in terms of \( u \) and \( v \). \[20 \text{ pts}\]

DO NOT EVALUATE.