Math 140, Jeffrey Adams/Test II SOLUTIONS

Question 1. (20 points) (a) 

\[ f'(x) = \frac{(\cos(2x) + x(-2\sin(2x)))e^{3x} - x\cos(2x)3e^{3x}}{e^{6x}} \]

Plugging in \( x = 0 \) gives \( f''(0) = \frac{(1+0)1-0}{1} = 1. \)

(b) 

\[ f'(x) = \frac{1}{\cos(x)}(-\sin(x)) = -\frac{\sin(x)}{\cos(x)} = -\tan(x), \]

and

\[ f''(x) = -\frac{\cos(x)(\cos(x) - \sin(x)(-\sin(x)))}{\cos^2(x)} = -\frac{1}{\cos^2(x)} = -\sec^2(x). \]

Question 2. (20 points)

(a) Differentiating gives \( \sin(x-y)(1-\frac{dy}{dx}) = 1 \). Solve this for \( \frac{dy}{dx} \):

\[ 1 - \frac{dy}{dx} = -\frac{1}{\sin(x-y)}, \]

or \( \frac{dy}{dx} = 1 + \frac{1}{\sin(x-y)}. \)

(b) Plugging in \( x = 0, y = \pi/2 \) gives \( \frac{dy}{dx} = 1 + \frac{1}{\sin(-\pi/2)} = 1 - 1 = 0. \)

(c) Since the the tangent line has slope 0 by (b), it is horizontal, with equation \( y = \pi/2. \) Alternatively, \( y = \frac{\pi}{2} = 0(x-0), \) or \( y = -\frac{\pi}{2}. \)

Question 3. (20 points)

Let \( O \) be the center of the circle, \( P \) be the point on the circle. Let \( L \) be a horizontal line through the center of the circle. Let \( \theta \) be the angle between the line \( OP \) and \( L \), and let \( h \) be the height of \( P \) above the ground.

Let \( \frac{dh}{dt} = 3, \) and we are looking for \( \frac{dh}{dt}. \) The equation relating them is \( h = 100\sin(\theta). \) Therefore \( \frac{dh}{dt} = 100\cos(\theta)\frac{d\theta}{dt} = 300\cos(\theta). \)

We need to find \( \cos(\theta) \) when \( h = 50. \) When \( h = 50, \) \( \sin(\theta) = 50/100 = \frac{1}{2}. \)

Therefore \( \cos(\theta) = \sqrt{1 - \sin^2(\theta)} = \sqrt{1 - \frac{1}{4}} = \sqrt{3}/2. \) Therefore \( \frac{dh}{dt} = 300\sqrt{3}/2 = 150\sqrt{3} = 259.8. . . . \)

Question 4. (20 points)

(a) Let \( f(x) = \ln(x), a = 20 \) and \( h = .01. \) Then \( f(20.01) \approx f(20) + (.01)f'(20). \)

Since \( f'(x) = \frac{1}{x}, f'(20) = \frac{1}{20} = .05. \) Therefore

\[ f(20.01) \approx 2.9957322735 + (.01)(.05) = 2.9962322735. \]

(b) The error is \( 2.9962322735 - 2.9962321485 = .0000001249 \) (or \( .0000001250, \) depending on how your round off.)

Question 5. (20 points)

The first step is \( c_1 = 4 - \frac{f(4)}{f'(4)} = 4 - \frac{\log(4) - 4 + 3}{\frac{4}{4-1}} = 4.5150591481 \) and the second is

\[ c_2 = 4.5150591481 - \frac{f(4.5150591481)}{f'(4.5150591481)} = 4.5150591481 - \frac{\log(4.5150591481) - 4.5150591481 + 3}{\frac{1}{4.5150591481} - 1} = 4.5052445368. \]