Math/Cmsc 475, Jeffrey Adams
Twelve-Fold Way

The number of ways of putting m balls into n boxes,

m balls	n boxes	no constraint	≤ 1	≥ 1
distinct	distinct	n^m	$n!/(n-m)!$	$n!S(m,n)$
identical	distinct	$\binom{n+m-1}{n-1}$	$\binom{n}{m}$	$\binom{n-1}{m-1}$
distinct	identical	$S(m,1)+S(m,2)+\cdots+S(m,n)$	$\begin{cases} 0 & m \leq n \\ 1 & m > n \end{cases}$!	$S(m,n)$
identical	identical	$p_1(m)+p_2(m)+\dots p_n(m)$	$\begin{cases} 0 & m \leq n \\ 1 & m > n \end{cases}$	$p_m(n)$

Notation:

1. $\binom{a}{b} = \frac{a!}{b!(a-b)!}$
2. $S(m,n)$ is a Stirling number of the second kind: the number of partitions of an m-set into n parts.
3. $p_k(m)$: number of partitions of m into k parts

See *Enumerative Combinatorics* by Richard Stanley.