MATH808O: Motivic Homotopy Theory-Spring 2023

A PDF version of the syllabus is available here.

Meeting times: MWF, 9:00am-9:50am (PHY 2106). There are a few days during the semester when Dr. Rosenberg will be unavailable at the regular time, and on these days, there will be a video recording in place of a regular in-person lecture. These days include February 8 and 10 (Dr. Rosenberg needs to be at a panel meeting at NSF), April 7 and April 12 (Passover), and April 28 and May 1. See the Canvas calendar for the plan of the lectures.

Instructor: Professor Jonathan Rosenberg. His office is room 2114 of the Math Building, phone extension 55166, or you can contact him by email. His office hours are M and W 10-11, or by appointment.

Text: *Motivic Homotopy Theory* by Dundas, Levine, Østvaer, Röndigs, and Voevodsky, Universitext, Springer, 2007. The ebook is available free through the university's Springer site license. Log in at **lib.umd.edu** to get it.

Other books on the subject:

- Vladimir Voevodsky, Andrei Suslin, Eric M. Friedlander, *Cycles, Transfers, and Motivic Homology Theories*, Annals of Math. Studies, no. 143, Princeton Univ. Press, 2000.
- Fabien Morel, *Introduction to $\mathbf{A}^1$-homotopy theory*, in ICTP Lect. Notes, XV, Abdus Salam Int. Cent. Theoret. Phys., Trieste, 2004.
- Fabien Morel, *Homotopy Theory of Schemes*, translated from the 1999 French original by James D. Lewis, Amer. Math. Soc., 2006.
- Carlo Mazza, Vladimir Voevodsky, and Charles Weibel, *Lecture notes on motivic cohomology*, Clay Mathematical Monographs, 2, Amer. Math. Soc. 2006.
- Marc Levine and Fabien Morel, *Algebraic Cobordism*, Physica-Verlag, 2007 and Springer, 2010.
- Fabien Morel, $\mathbf{A}^1$*-Algebraic Topology Over a Field*, Lecture Notes in Math., vol. 2052, Springer, 2012.

For a quick introduction to the subject, you might start with Voevodsky's 1998 ICM lecture, even though it's been superseded. You can find it in the **Files** section of the course website on Canvas or else here. Also in the **Files** section of the course website on Canvas you will find many of the foundational papers on the subject and references for which UMd has a site license.

Prerequisite: Algebraic geometry (MATH 606) and algebraic topology (MATH 734) (or their equivalents) are preferred, but you might be able to get by with only one of these if you know some concepts from the other.

Course Description:

Motivic homotopy theory, which grew out of earlier work of Suslin and Voevodsky on "motivic cohomology," was invented by Voevodsky and Morel as a way of importing techniques from algebraic topology to solve problems in algebraic geometry. In this it has been remarkably successful, and led to Voevodsky's proof of the Milnor conjecture and his Fields Medal. But new developments in the last twenty years have gone much further, and now the subject has been applied to enumerative algebraic geometry (over general fields and not just over the complex numbers) and to classical algebraic topology as well, for example, almost doubling the range in which one can compute the stable homotopy groups of spheres. So motivic homotopy theory is now one of the most active areas in *both* algebraic geometry and algebraic topology. I hope to give you a bit of a feel for why the subject is important and a few of the main ideas, even though the machinery to master it is quite formidable and can't be covered in one semester.

Course Requirements:

There will be a few problem sets for you to work on during the semester. In addition, each enrolled student will be asked present a paper of his/her choice (related to the course material) to the rest of the class. I'll provide lots of suggestions and help for this.