I am quite grateful to those who have sent me their comments on this book. I especially thank Paul Arne Ostvaer, Ioannis Emmanouil, Desmond Sheihah, Efton Park, Jon Berrick, Henrik Holm, and Hanfeng Li for their corrections.

Chapter I

page 11. In Exercise 1.2.9, part (2), the condition should read: . . . all the entries of the matrix $A$ are non-negative and no column of $A$ is identically 0.

page 25, line 3. Change “algebraic closure” to “integral closure”.

page 26, bottom line. Change $p$ to $\pm p$.

page 39, second paragraph. One small technical point: the monoid described in Exercise 1.1.7 really corresponds to the monoid of isomorphism classes of oriented vector bundles over $S^2$. The problem stems from the fact that $O(r)$ is disconnected and from the fact that $\pi_{n-1}(O(r))$ is defined using based maps from $S^{n-1}$ to $O(r)$, which necessarily land in the connected component of the identity. This only makes a difference in the classification of rank-2 bundles, where the oriented bundles corresponding to integers $m$ and $-m$ in $\mathbb{Z}$ are isomorphic as unoriented bundles. So to get the corresponding description of the monoid of unoriented bundles, one should replace the condition $m \in \mathbb{Z}$ by $m \in \mathbb{N}$ for $n = 2$.

Chapter II

page 65, lines 5 and 6 from bottom. This argument isn’t correct when $R$ is noncommutative, since in that case $(AB)^t$ may not be equal to $B^tA^t$, and in fact property (f) in the Theorem should have been restricted to the commutative case. Here is a counterexample with $R = \mathbb{H}$, the quaternions. Note that in this case $R^*_{ab} = \mathbb{R}^*_+$, the positive reals, since the group of unit quaternions (isomorphic to the Lie group $SU(2)$) is its own commutator subgroup. So the map $a \mapsto \bar{a}$ is just the usual absolute value for quaternions.

Let $n = 2$ and

$$A = \begin{pmatrix} 2 & i \\ j & k \end{pmatrix}, \quad \text{so} \quad A^t = \begin{pmatrix} 2 & j \\ i & k \end{pmatrix}.$$  

Subtracting $\frac{j}{2}$ times the first row of $A$ from the second row changes the second row to $(0 \ k - \frac{j}{2} i)$ and thus results in a $\frac{1}{2}k$ in the lower right. So $\det A = 2 \cdot \frac{1}{2} = 3$. On the other hand, subtracting $\frac{i}{2}$ times the first row of $A^t$ from the second row changes the second row to $(0 \ k - \frac{1}{2} j)$ and thus results in a $\frac{1}{2}k$ in the lower right. So $\det A^t = 2 \cdot \frac{1}{2} = 1$ and $\det A \neq \det A^t$.

page 68, middle. The proof for the case $b_k \in R^*$, $b_i \in \text{rad} \ R$ for $i \neq k$, is incomplete. It should really read as follows:
In this case,

\[
\det_n A' = (-1)^{k-1} \left( \frac{b_k'}{b_k} \right)^{-1} \det_{n-1} \left( \begin{array}{c} B_1 \\ \vdots \\ B_i + aB_k \\ \vdots \\ \tilde{B}_k \\ \vdots \\ B_n \end{array} \right),
\]

and

\[
\det_n A = (-1)^{k-1} \bar{b}_k^{-1} \det_{n-1} \left( \begin{array}{c} B_1 \\ \vdots \\ B_i \\ \vdots \\ \bar{B}_k \\ \vdots \\ B_n \end{array} \right).
\]

But

\[
B_i + aB_k = B_i - a \sum_{j \neq k} b_k^{-1}b_jB_j = (1 - ab_k^{-1}b_i)B_i - a \sum_{j \neq i, k} b_k^{-1}b_jB_j,
\]

so by properties (a) and (c) for \( \det_{n-1} \), we have

\[
\det_n A' = (-1)^{k-1} \left( \frac{b_k'}{b_k} \right)^{-1} \left( 1 - ab_k^{-1}b_i \right) \det_{n-1} \left( \begin{array}{c} B_1 \\ \vdots \\ B_i \\ \vdots \\ \bar{B}_k \\ \vdots \\ B_n \end{array} \right).
\]

Thus to show that \( \det_n A' = \det_n A \), it suffices to show that

\[
(b_k - b_i a)^{-1} \left( 1 - ab_k^{-1}b_i \right) = \bar{b}_k^{-1}
\]

in \( R_{ab}^\times \), or that

\[
(1 - ab_k^{-1}b_i)b_k \equiv b_k - b_i a \text{ in } R^\times \text{ mod } [R^\times, R^\times].
\]

If we factor the right-hand side as \( b_k(1 - b_k^{-1}b_i a) \) and let \( u = b_k^{-1}b_i \in \text{rad } R \), then it suffices to show that

\[
1 - au \equiv 1 - ua \text{ in } R^\times \text{ mod } [R^\times, R^\times].
\]
There are now two cases. If \( a \in R^\times \), then \( 1 - au = a(a^{-1} - u) \) while \( 1 - ua = (a^{-1} - u)a \), and \( (a^{-1} - u) \in R^\times \) since \( u \in \text{rad} \, R \). So this case is clear. If, on the other hand, \( a \not\in R^\times \), then \( (1 - a) \in R^\times \), \( v = 1 + u + ua \in R^\times \), and

\[
(1 - au)(1 - a) = (1 - a - au + uau) = 1 - av,
\]

\[
(1 - ua)(1 - a) = (1 - a - ua + ua^2) = 1 - va,
\]

so \( (1 - au)(1 - ua)^{-1} = (1 - av)(1 - va)^{-1} \)

\[
= (v^{-1} - a)v(v^{-1} - a)^{-1}v^{-1} \in [R^\times, R^\times].
\]

So this confirms property (a) for \( \det_n \).

page 69, bottom line. The last sentence should say: Deduce that \( e^{2j\theta_1}, e^{2k\theta_2}, \) and \( e^{2i\theta_3} \) are all commutators.

page 76, line 10. Change “One” to “Once”.

page 80, line 6. Change \( R^{r_1 + r^2} \) to \( R^{r_1 + r_2} \).

page 82, line 15. Change \( E_1 \) to \( E_2 \).

page 91, third line from bottom. This should read: Suppose \((C_1, d_1)\) and \((C_2, d_2)\) are complexes . . . .

page 104, line 6. The displayed equation should read

\[
[a \ b]_I^k = [a \ b^k]_I = 1.
\]

Chapter III

page 149, 9th line from bottom. The equation should read \( A'T = TA \), not \( A'T = TP \).

page 158. In Exercise 3.3.6, the correct statement of part (1) is as follows: Fill in the details of the argument copied from Theorem 1.3.11, that if \( S \) is a ring and if \( J \) is an ideal of \( S \) contained in \( \text{rad} \, S \), then the map \( K_0(S) \to K_0(S/J) \) induced by the quotient map \( S \to S/J \) is injective.

Chapter IV

page 184. The matrix identity in part (1) of Exercise 4.1.28 was supposed to read:

\[
\begin{pmatrix}
d & 0 \\
0 & d^{-1}
\end{pmatrix}
\begin{pmatrix}
1 & a \\
0 & 1
\end{pmatrix}
\begin{pmatrix}
d^{-1} & 0 \\
0 & d
\end{pmatrix}
\begin{pmatrix}
1 & -a \\
0 & 1
\end{pmatrix}
= \begin{pmatrix}
1 & (d^2 - 1)a \\
0 & 1
\end{pmatrix}
\]

page 189. In the proof of Theorem 4.2.4, there is a sentence missing two lines up from the end of the proof. Before the sentence “Since these generate . . .” one should insert the line: Similarly, \( x \) commutes with \( x_{Nk}(a) \) for \( N \) large enough, for any \( k < N \), and for any \( a \in R \).

page 190. The beginning of the proof of Theorem 4.2.7 should say: “By Theorem 4.2.4”.

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page 191, line 5 from the bottom. This should read \( s : \text{St}(R) \to U \).

page 237. There are some misprints in the last sentence of the statement of Theorem 4.4.19. The sentence should read: “Similarly, if \( \mathbb{F} = \mathbb{C} \), \( K_0(R) \cong KU^0(X) \), \( K_1(R) \) is an extension of \( KU^{-1}(X) \) by the connected component of the identity in \( R^\times \), and \( K_2(R) \) surjects onto \( \widetilde{KU}^{-2}(X) \).”

page 244, lines 6 and 7 from the bottom. The phrase “with the map \( K_1(R[s, t], (st - 1)) \to K_1(R) \) induced by mapping \( R[s, t] \) to \( R \)” should be replaced by “with a certain map \( K_1(R[s, t], (st - 1)) \to K_1(R) \).” (The obvious map is 0, but there is another map that can be defined with more work.)

\[ \cdots \to K_{i+1}(R/I) \xrightarrow{\partial} K_i(R, I) \to K_i(R) \to K_i(R/I) \xrightarrow{\partial} K_{i-1}(R, I) \to \cdots \]

page 258. The statement of part (1) of Proposition 5.1.20 should say: The projection \( p : E = B \times F \to B \) onto the first factor . . .

page 266, line 14. The displayed equation should read:

\[ H_\bullet(\widetilde{X}^+, \widetilde{X}; \mathbb{Z}) = 0. \]

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