APPLICATIONS OF EXPONENTIAL FUNCTIONS

Exponential Growth Model: $A = A_0 e^{kt}$. **(k >0)**

Exponential Decay Model: $A = A_0 e^{kt}$. **(k <0)**

In both of these models ***k* represents the growth rate (the growth constant)** and $\mathbf{A_0}$ **represents the initial amount**, i.e. the value of A (the number of bacteria present, the amount of radioactive material, the amount in the account etc.) when $t = 0$. So, let $t = 0$ and compute the value of A:

$A = A_0 e^{k(0)} = A_0 e^0 = A_0(1) = A_0$. Therefore, A_0 **is the amount when *t* = 0, i.e. the initial amount.**

Note that this model is the same model as the one used for compounding interest continuously, $A = Pe^{rt}$. Here the growth rate is the interest rate, represented by *r* instead of by *k* as in the models above. A_0 is represented by *P*, the principal, or the **initial amount** in the account.

We use t as the independent variable here because in most models dealing with exponential functions the variable represents time.

Note that if we want to determine the time it will take for an amount A in an account to **double**, we need to find the value of t such that the value of A is twice the initial amount, i.e. such that $A = 2A_0$. So, replace A with $2A_0$ and solve for t. If we want the time it will take for the initial amount to **triple**, find A such that $A = 3A_0$.

Exercises

1. ***Interest compounded continuously*** $4000 is invested in an account at an annual rate of 4% interest. Determine (a) the amount in the account after 8 years, (b) how many years it will take for the money in the account to double, and (c) how long will it take for the money to triple.

2. ***Bacteria Growth*** The number of bacteria in a culture after *t* hours is given by the model $A = 300e^{kt}$. After 5 hours the bacteria count is 980. Determine (a) the initial amount of bacteria in the culture, (b) the value of the growth constant *k*, (c) how long it will take the bacteria count to double, and (d) how long it will take the count to triple.

3. ***Population Growth*** The population of a city grows according to the model $P = 170{,}000e^{.022t}$. If $t = 0$ corresponds to the year 2000, determine the year during which the population will be 250,000 .

4. ***Radioactive Decay*** The radioactive isotope radium has a half-life of 1620 years. An amount of radium decays according to the model $A = A_0 e^{kt}$. If the initial amount present is 14 grams, determine (a) the amount of radium that is left after 1620 years (do it the easy way!), and (b) the amount that is left after 500 years. (Recall that the statement “the half-life is 1620 years” means that after 1620 years, ½ of the initial amount remains).

5. ***Radioactive Decay*** The radioactive isotope carbon has a half-life of 5730 years. After 10,000 years 2.4 grams remain. Determine the initial amount of carbon that was present.