# Solutions: Applications of Exponential Functions

1. The model is: $A = 4000e^{0.04t}$

   (a) To find the amount after 8 years, let $t = 8$: $A = 4000e^{0.04(8)} = 4000e^{0.32} \approx \$5508.51$

   (b) We need to find $t$ such that there is 2(4000), or \$8000 in the account. So, substitute 8000 for $A$: $8000 = 4000e^{0.04t} \Rightarrow 2 = e^{0.04t}$

   $$\ln 2 = \ln e^{0.04t}$$
   $$\ln 2 = 0.04t$$
   $$t = \frac{\ln 2}{0.04} \approx 17.33 \text{ years}$$

   (c) Find $t$ such that we have 3(4000) or \$12,000 in the account (Let $A = 12{,}000$):

   $$12000 = 4000e^{0.04t} \Rightarrow 3 = e^{0.04t}$$
   $$\ln 3 = \ln e^{0.04t}$$
   $$\ln 3 = .04t\text{, so: } t = \frac{\ln 3}{0.04} \approx 27.47 \text{ years}$$

2. (a) From the given model we can see that $A_0$ is 300. Therefore, *the initial bacteria count is 300.*

   (b) We are given: after 5 hrs. the count is 980. So, when $t = 5$, $A = 980$:

   $980 = 300e^{k(5)}$ We can now find the growth constant $k$:

   $$\frac{980}{300} = e^{5k}$$
   $$\ln\left(\frac{98}{30}\right) = \ln e^{5k}$$
   $$\ln\left(\frac{98}{30}\right) = 5k$$
   $$k = \frac{\ln\left(\frac{98}{30}\right)}{5} \approx 0.236754$$

   (c) We need to find $t$ such that the bacteria count is twice the initial amount, or 2(300) or 600, So:

   $$600 = 300e^{0.236754t}$$
   $$2 = e^{0.236754t}$$
   $$\ln 2 = \ln e^{0.236754t}$$
   $$\ln 2 = 0.236754t$$
   $$t = \frac{\ln 2}{0.236754} \approx 2.93$$ hours is the time it will take the bacteria count to double.

   (d) Find $t$ such that the count is 3(300) or 900: $900 = 300e^{0.236754t} \Rightarrow 3 = e^{0.236754t}$

   $$\ln 3 = \ln e^{0.236754t}$$
   $$\ln 3 = 0.236754t$$
   $$t = \frac{\ln 3}{0.236754} \approx 4.64$$ *hours to triple.*

3. Find $t$ such that $P = 250{,}000$: $250{,}000 = 170{,}000e^{0.022t}$

   $$\frac{25}{17} = e^{0.022t}$$
   $$\ln\left(\frac{25}{17}\right) = \ln e^{0.022t}$$
   $$\ln\left(\frac{25}{17}\right) = 0.022\,t$$
   $$t = \frac{\ln\left(\frac{25}{17}\right)}{0.022} \approx 17.53 \text{ years}$$

*So during the year 2017 the population wil be 250,000.*

**4.** (a) Since the half-life of radium is 1620 years, we know that after 1620 years 1/2 of the initial amount is left. We are given that the initial amount of radium is 14 grams. Therefore, after 1620 years 1/2(14) or 7 *grams* is left.

(b) We need to find $A$ when $t = 500$. Note that since we are given the initial amount, 14 grams, we let $A_0 = 14$: $A = 14e^{k(500)}$. We want to solve for $A$, but we can't until we know the value of $k$. So, we'll use the fact thet the 1/2 life is 1620 years to find $k$. When $t = 1620$, the amount, $A$, is half the intial amount, or 1/2(14) or 7:

So: $7 = 14e^{k(1620)} \Rightarrow 0.5 = e^{1620k}$

| | |
|---|---|
| $\ln 0.5 = \ln e^{1620k}$ | $k = \dfrac{\ln 0.5}{620} \approx$ |
| $\ln 0.5 = 1620k$ | $-0.000427869$ |

(Note that $k$ is negative. This makes sense b/c we're dealilng with exponential decay)

Now we can find the amount that is left when $t = 500$:

$A = 14e^{-0.000427869(500)} \approx 11.3039$ *grams.*

**5.** We Know from the information given that when $t = 10{,}000$, $A = 2.4$ grams. We want to find $A_0$.

We have: $2.4 = A_0e^{k(10{,}000)}$. We must first find $k$. Since the 1/2 lilfe of carbon is 5730 years,

when $t = 5730$, $A = \left(\frac{1}{2}\right)A_0$ So: $\left(\frac{1}{2}\right)A_0 = A_0e^{k(5730)}$ Diving both sodes by $A_0$ we have:

$\frac{1}{2} = e^{5730k}$ Now we can solve for $k$:

$\ln 0.05 = \ln e^{5730k}$

$\ln 0.05 = 5730k$, so $k = \dfrac{\ln 0.5}{5730} \approx -0.000120968$

Now we can find $A_0$: $2.4 = A_0e^{-0.000120968}(10{,}000)$

$$A_0 = \frac{2.4}{e^{-0.000120968(10{,}000)}} \approx 8.0458$$ *grams, the initial amount of carbon.*