SOLUTIONS TO THE WORKSHEET, SECTION 3.1:

APPLICATIONS OF QUADRATIC FUNCTIONS
MAXIMUM AND MINIMUM VALUES

1. (a) $P = 600 \Rightarrow 2x + 2y = 600.$ Solving for y: $2y = 600 - 2x, \quad so\ y = 300 - x$.

   Area = lw, so: $A = xy \Rightarrow A = x(300 - x)$. So $\underline{A = 300x - x^2}$

   (b) We need to find the x-coordinate of the vertex of the above quadratic function.
   $x - \dfrac{-b}{2a} = \dfrac{-300}{2(-1)} = \dfrac{300}{2} = 150$. So, if x, the width, is 150 feet we will have the rectangle with the maximum possible area. If x = 150, $y = 300 - 150 = 150$
   <u>Therefore, the dimensions of the rectangle with the maximum possible area are: 150' by 150'</u>

2. (a) $P = 920 \Rightarrow 4x + 2y = 920$. Solving for y: $2y = 920 - 4x$, so $y = 460 - 2x$.
   Area = lw, so: $A = xy = x(460 - 2x)$. So $\underline{A = 460x - 2x^2}$

   (b) We need the x-coordinate of the vertex of the quadratic function representing the area. (NOTE: If we were asked for the maximum possible area as opposed to the dimensions of the corral with max area, we would need to find the y-coordinate of the vertex.)
   $x = \dfrac{-b}{2a} = \dfrac{-460}{2(-2)} = \dfrac{460}{4} = 115$. If x is 115 feet, then
   $y = 460 - 2x = 460 - 2(115) = 230$. <u>So, the dimensions that will yield the corral with the maximum possible area are: 115 ft by 230 ft.</u>

3. (a) We want to find the x-coordinate of the vertex. $x = \dfrac{-b}{2a} = \dfrac{-(-30)}{2(0.3)} = \dfrac{30}{.6} = \dfrac{300}{6} = 50$
   Therefore, producing 50 units per day will result in the minimum possible cost.

   (b) Now we need the y-coordinate of the vertex:
   $y = f(\dfrac{-b}{2a}) = f(50) = 900 - 30(50) + .3(50)^2 = 150$. So, the minimum daily cost is $150.

4. (a) We need the x-coordinate of the vertex:
   $x = \dfrac{-b}{2a} = \dfrac{-120}{2(-.0003)} = \dfrac{120}{.0006} = \dfrac{1200000}{6} = 200{,}000.$ So producing 200,000 units per day will yield the maximum possible profit.

   (b) We need the y-coordinate of the vertex:

   $$y = f(\frac{-b}{2a}) = f(200{,}000) = -.0003(200{,}000)^2 + 120(200{,}000) - 180{,}000 = \$11{,}820{,}000$$
   , the maximum possible profit.