Math 246H Laplace Transform Problems

Problem 1: For each of the differential equations below, find the Laplace transform $Y(s) = \mathcal{L}\{y(t)\}$.

1. $y'' + 3y' + 2y = 0 \quad y(0) = 1 \quad y'(0) = 0 \quad \text{[Answer: } Y(s) = \frac{s+3}{s^2+3s+2}\]$

2. $y'' - 2y' + 2y = \sin(3t) \quad y(0) = 1 \quad y'(0) = 0 \quad \text{[Answer: } Y(s) = \frac{s^3 - 2s^2 + 9s - 15}{(s^2+9)(s^2-2s+2)}\]$

3. $y'' + 2y' + y = 4e^{-t} \quad y(0) = 2 \quad y'(0) = -1 \quad \text{[Answer: } Y(s) = \frac{2s^2+3s+5}{(s+1)(s^2+2s+1)}\]$

Problem 2: For each function below, find the inverse transform $y(t) = \mathcal{L}^{-1}\{Y(s)\}$ and sketch the graph of $y(t)$.

1. $Y(s) = \frac{2e^{-2s}}{s^2+4} \quad \text{[Answer: } y(t) = u_2(t) \sin(2(t - 2))\]$

2. $Y(s) = \frac{e^{-s}}{(s-3)^2+4} \quad \text{[Answer: } y(t) = \frac{1}{2}u_1(t)e^{3(t-1)} \sin(2(t - 1))\]