Trivial and Vacuous Proofs

1. **Trivial Proofs**:

   **Goal**: To prove the statement:
   
   \[
   \text{let } x \in S. \text{ If } P(x) \text{ then } Q(x).
   \]

   To do this, we show that \(Q(x)\) is true for all \(x \in S\).

   **Example**: Let \(x \in \mathbb{R}\). If \(x < 5\), then \(x^2 - 1 > -3\).

   **Proof**: Let \(x \in \mathbb{R}\). We want to show that \(x^2 - 1 > -3\) for every \(x \in \mathbb{R}\). Recall that \(x^2 \geq 0\) \(\forall x \in \mathbb{R}\). Thus,
   
   \[
   x^2 - 1 \geq 0 - 1 = -1 > -3.
   \]

   Hence the implication is true. \(\blacksquare\)

2. **Vacuous Proofs**:

   **Goal**: To prove the statement:
   
   \[
   \text{let } x \in S. \text{ If } P(x) \text{ then } Q(x).
   \]

   To do this, we show that \(P(x)\) is false for all \(x \in S\).
Ex: Let $x \in \mathbb{R}$. If $x^2 + 1 \leq 0$ then $x^3 \geq 8$.

Proof: Let $x \in \mathbb{R}$. We want to show that $x^2 + 1 \leq 0$ is false for every $x \in \mathbb{R}$. Notice that $x^2 \geq 0$ for every $x \in \mathbb{R}$. Thus $x^2 + 1 \geq 0 + 1 = 1$.

Hence the implication is true. ♡