MATH/AMSC 673 (Fall 2015)
PARTIAL DIFFERENTIAL EQUATIONS I
Tu-Th 5:00-6:15, MTH 1311

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Prerequisites: MATH 411 or equivalent.

Objectives: This course is a first semester of a two semester sequence. It is an introduction to the classical theory of PDE. The focus will be on developing basic calculus ideas for prototype second order linear PDE, that is the Laplace, heat, and wave equations, as well as first order nonlinear PDE. The second semester will treat modern methods for PDEs (distributions, functional analysis, Sobolev spaces, bounded and compact operators in Hilbert spaces).

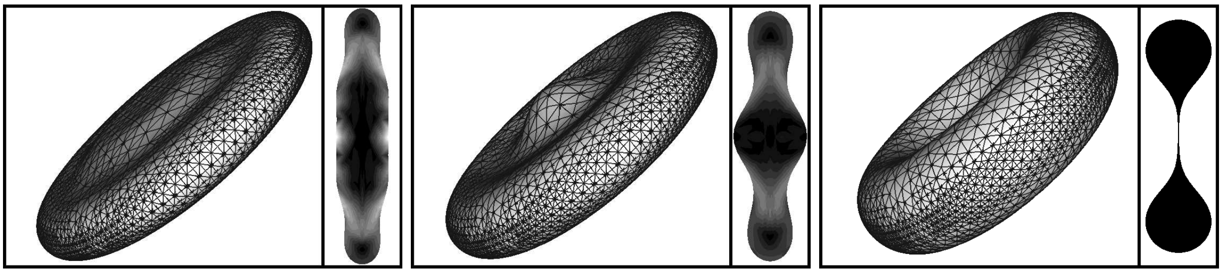

Figure 1: Evolution of a fluid biomembrane with initial axisymmetric ellipsoidal shape of aspect ratio $5 \times 5 \times 1$ and final shape similar to a red blood cell. Each frame shows the membrane mesh and a symmetry cut along a big axis. The fluid flow is quite complex, creating first a bump in the middle and next moving towards the circumference and producing a depresion in the center with flat pinching profile.

Course Outline

1. Introduction to PDE [1 and notes]

- Derivation of basic PDEs: transport equation, conservation of mass, momentum and energy, heat and wave equations, Laplace equation, minimal surfaces, Hamilton-Jacobi
- Classification of PDE

2. Laplace's Equation [1,3]

- Dirichlet and Neumann boundary conditions
- Fundamental solution
- Mean value property
- Harnack's inequality
- Regularity of harmonic functions
- Green's functions
- Strong and weak maximum principle - Perron method

- Energy methods
- Calculus of variations, Lax-Milgram
- Finite difference and finite element methods

3. Heat Equation [1]

- Initial boundary value problems
- Fundamental solution, Duhamel's principle
- Strong and weak maximum principle
- Energy methods

4. Wave Equation [1]

- D'Alembert's formula
- Sprerical means ($n = 3$), method of descent ($n = 2$)
- Huygen's phenomenon, finite speed of propagation
- Duhamel's principle
- Energy method

4. First Order Nonlinear PDE [1,2]

- Characteristics: characteristic curves, systems of quasi-linear PDE, local existence
- Hamilton-Jacobi equations
- Conservation laws: scalar laws, p-system, characteristics, Riemann invariants, singularities
- Weak solutions, Rankine-Hugoniot jump conditions, nonuniqueness
- Shock waves, Lax shock conditions
- Shock and rarefaction waves for systems, Riemann problems
- Entropy conditions, viscosity solutions

Texts.

[1] L.C. Evans, *Partial Differential Equations*, Graduate Studies in Mathematics Vol 19, AMS (1998), ISBN 0-8218-0772-2.

[2] M. Renardy and R.C. Rogers, *An Introduction to Partial Differential Equations*, Texts Appl. Math. 13, Springer, 1993.

[3] D. Gilbarg and N.S. Trudinger, *Elliptic Partial Differential Equations of Second Order*, Springer (1983).