(1) [4] For each of the following ordinary differential equations, give its order and state whether it is linear or nonlinear.

(a) \( \frac{d^3w}{dx^3} + w^2 \frac{dw}{dx} + e^x w = x^2; \)  
   Solution. third order, nonlinear.

(b) \( \frac{d^5y}{ds^5} = 2s \frac{d^2y}{ds^2} + \sin(s). \)  
   Solution. fifth order, linear.

(2) [4] Solve the initial-value problem

\[ tz' + 3z + 2t = 0, \quad z(1) = 0. \]  

Solution. This equation is linear. Its normal form is

\[ \frac{d}{dt} (t - 3z) = t^{-3} \cdot 2 = 2t^{-3}. \]

Integrating both sides yields

\[ t^{-3}z = -t^{-2} + c. \]

Imposing the initial condition gives

\[ 1^{-3} \cdot 0 = -1^{-2} + c, \]

whereby \( c = 1^{-2} = 1. \) The solution is therefore

\[ z = -t + t^3, \quad \text{for every } t > 0. \]

Remark. The interval of definition for this solution is \((0, \infty)\). Can you see why?

(3) [2] Give the interval of definition for the solution of the initial-value problem

\[ \frac{dx}{dt} + \frac{1}{t^2 - 1} x = \frac{1}{\sin(t)}, \quad x(2) = -3. \]

(You do not have to solve this equation to answer this question!)

Solution. This equation is linear and is already in normal form. The coefficient \( 1/(t^2 - 1) \) is continuous everywhere except at \( t = \pm 1 \), while the forcing \( 1/\sin(t) \) is continuous everywhere except where \( t = n\pi \) for some integer \( n \). You can therefore read off that the interval of definition for its solution is \((1, \pi)\) because:

- the initial time \( t = 2 \) is in \((1, \pi)\),
- the coefficient and forcing are both continuous over \((1, \pi)\),
- the coefficient is not defined at \( t = 1 \),
- the forcing is not defined at \( t = \pi \).