AMSC 466. Spring 2020

Homework Assignment 2. Due Thursday Feb. 20.

1. **5 pts** Consider the iteration

$$x_{k+1} = T(x_k) := 2x_k - 3x_k^2.$$

   (a) What are the fixed points of this iteration?

   (b) Find the maximal interval where the map $x_{k+1} = T(x_k)$ is a contraction.

   (c) What is the order of convergence to the nonzero fixed point x^*?

   (d) For which values of x_0 does this iteration converge to x^*?

2. **5 pts**

   (a) Consider the equation $x^m = 0$ where m is an integer greater than 1. Then $x^* = 0$ is the only root, and it is degenerate. Show that the Newton's method converges linearly and find the the asymptotic error constant.

   (b) Let f be a real-valued function of one variable continuous derivatives of all orders. Let $f(x^*) = 0$, $f'(x^*) \neq 0$, $f''(x^*) = 0$, and $f'''(x^*) \neq 0$. Prove that then the Newton iteration converges cubically (i.e. with order 3).

3. **10 pts**

   The Van der Pol oscillator

$$\begin{aligned} \dot{y_1} &= y_2, \\ \dot{y_2} &= \mu(1 - y_1^2)y_2 - y_1 \end{aligned} \tag{1}$$

   has a stable and globally attracting periodic solution for each $\mu \geq 0$. The Matlab code `FindPeriodicSolution.m` finds the periodic solution for the given μ and returns the maximal value of y for this solution. Consider the problem of finding μ such that the maximal value of y for this periodic solution is 10. The Matlab command setting up the corresponding nonlinear equation is:

```
fun = @(x)FindPeriodicSolution(x) - 10;
```

   The code `SolveNonlinEq.m` solves this equation using the bisection method (`method = 1`) and a quasinewton method with the derivative approximated using the forward difference (`method = 2`).

   Read Lecture 5 in G. W. Stewart, Afternotes on Numerical Analysis, implement the hybrid method (secant/bisection) described there, and use it to solve this nonlinear equation. Make your code print out the iteration number and the corresponding estimate for the solution μ at each iteration.

   Submit a printout of your implementation of the hybrid method, a printout of your code. Comment out all plotting commands, and write a report comparing **the numbers of iterations** *and* **runtimes** *in the bisection method, quasinewton method with the forward difference for derivative estimation, and the hybrid method.*