Inverse Problem in Seismic Imaging

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We address the problem of estimating sound speeds (seismic velocities) inside the earth which is necessary for obtaining seismic images in regular Cartesian coordinates. The main goals are to develop algorithms to convert time migration velocities to true seismic velocities, and to convert time-migrated images to depth images in regular Cartesian coordinates.

Our main results are three-fold. First, we establish a theoretical relation between the seismic velocities and the time migration velocities using the paraxial ray tracing theory. Second, we formulate an appropriate inverse problem describing the relation between time migration velocities and depth velocities and show that this problem is mathematically ill-posed, i.e., unstable to small perturbations. Third, we develop numerical algorithms to solve regularized versions of these equations which can be used to recover smoothed velocity variations. Our algorithms consist of efficient time-to-depth conversion algorithms based on Dijkstra-like Fast Marching Methods, as well as level set and ray tracing algorithms for transforming Dix velocities into seismic velocities. Our algorithms are applied to both two-dimensional and three-dimensional problems and we test them on a collection of both synthetic examples and field data.

1 Introduction

Producing an accurate image of the Earth’s interior is a challenging aspect of oil recovery and earthquake analysis [9]. The ultimate computational goal, which is to accurately produce a detailed interior map of the Earth’s makeup on the basis of external soundings and measurements, is currently out of reach for several reasons. First, although vast amounts of data have been obtained in some regions, this has not been done uniformly, and the data contain noise and artifacts. Simply sifting through the data is a massive computational job. Second, the fundamental inverse problem, namely to deduce the local sound speeds of the earth that give rise to measured reflected signals, is exceedingly difficult. Shadow zones and complex structures can make for ill-posed problems, and require vast computational resources.

Nonetheless, seismic imaging is a crucial part of the oil and gas industry. Typically, one makes assumptions about the earth’s substructure (such as laterally homogeneous layering), and then uses this model as input to an iterative procedure to build perturbations that more closely satisfy the measured data [9]. Such models often break down when the material substructure is significantly complex [8]: not surprisingly, this is often where the most interesting geological features lie.

The two types of Earth imaging, namely “time migration” and “depth migration” [9], rely on different coordinate views. Time migration produces images in ”time coordinates” (if \( A \) is a subsurface point then its time coordinates \( (x_0, t_0) \) are the location where the spherical wave from \( A \) first hits the surface and the minimal travel time from it to the surface). The corresponding characteristic from the point \( A \), called an image ray [4], arrives normally at the surface point \( x_0 \). Time migration is a fast and efficient technique. However, it is adequate only if seismic velocities are almost horizontally constant. The other kind of earth imaging, depth migration, is adequate for arbitrary seismic velocity. However, in addition to the standard seismic data it also requires the seismic velocity (the sound speed inside the earth) as input. The output of depth migration is a seismic image in depth coordinates (if \( A \) is a subsurface point then its depth coordinates \( x \) are its lateral position and its depth).

2 Theoretical results

Our central theoretical results are the relations between time migration velocities and the true seismic velocities in 2D and 3D. They form the basis for our numerical schemes (see [1] for details).

Seismic velocities and time migration velocities are connected through the quantity \( Q \) which characterizes the degree of convergence or divergence of the image rays \([6], [2]\). \( Q \) is a scalar in 2D and a \( 2 \times 2 \) matrix in 3D. The simplest way to introduce \( Q \) is the following: take an image ray \( x(x_0, t) \), call it central, and trace it from the surface downward into the Earth. Consider a small tube of rays around it. All these rays start perpendicular to the surface from a small neighborhood \( dx_0 \) of the point \( x_0 \). Thus, they represent a fragment of a plane wave propagating downward. Consider the fragment of the wave front defined by this ray tube at a time \( t_0 \). Let \( dq \) be the fragment of the tangent to the front at the point \( x(x_0, t_0) \) (Fig. 1), bounded
by the ray tube. Then, in 2D, $Q(x_0, t_0) = \lim_{dx_0 \to 0} \frac{dq}{dx_0}$, and in 3D, $Q_{ij}(x_0, t_0) = \lim_{dx_0 \to 0} \frac{dq_i}{dx_0}$, $i, j = 1, 2$, where $dq_1$, $dq_2$ are the lengths of the tangent along certain mutually orthogonal directions [6], [2].

Surprisingly, our theoretical relation in 2D allowed us to show that the Dix velocities $v_{Dix}(x_0, t_0)$ [3], which are the conventional estimate of the seismic velocities $v(x, z)$ from the time migration velocities $v_m(x_0, t_0)$, can be used as a more convenient input for our inversion algorithms. In 2D, our result is

$$v_{Dix}(x_0, t_0) \equiv \sqrt{\frac{\partial}{\partial t_0} (t_0 v_m^2(x_0, t_0))} = \frac{v(x(x_0, t_0), z(x_0, t_0))}{|Q(x_0, t_0)|}.$$  

(1)

In 3D, the relation is similar:

$$\frac{\partial}{\partial t_0} (t_0 V_m^2(x_0, t_0)) = v^2(x(x_0, t_0)) \left( Q(x_0, t_0)^T Q(x_0, t_0) \right)^{-1},$$  

(2)

where $V_m$ is the matrix of the time migration velocities.

### 3 Numerical algorithms and examples

Although we proved that finding seismic velocities from Dix velocities is an ill-posed problem, we were, nonetheless, able to develop techniques which attempt the smoothed reconstruction [1]. The first approach consists of two algorithms: ray tracing in a medium with unknown velocity and an efficient time-to-depth conversion. The motivation and building block for the latter is Sethian’s fast marching method [7]. Our time-to-depth conversion algorithm simultaneously solves two coupled equations: the Eikonal equation for first arrival propagating waves in a medium with unknown speed/cost function, and an auxiliary orthogonality relationship for the time coordinates. We were able to extend this approach to 3D. The second approach is based on the level set method [5]. This less efficient method can successfully handle the collisions of the rays following the first arrival front.

We tested our algorithms on a collection of synthetic examples and applied them to field data examples. Here we include one of the synthetic examples to provide a comparison between the results of our inversion and the conventional Dix inversion (Fig. 2).

### 4 Conclusions

We have derived theoretical relations between the seismic velocities and the migration velocities. We have posed an inverse problem of estimating seismic velocities from the time migration velocities and developed two numerical approaches for solving it. We have demonstrated that our velocity estimate is much more accurate than the conventional Dix estimate. Moreover, as our example shows, the Dix estimate may differ qualitatively from the true seismic velocity.

### References


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