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The main theorem of the paper (Theorem 1) is true as stated. However, the proof requires a modification. We thank Lili He for bringing this to our attention.

Theorems 2 and 2.2 are not true as stated for null forms involving time derivatives, such as \( Q_{0i} \) and \( Q_0 \). They have to be modified by including the terms \( \|F\|_{L^2(dt)L^3(dx)} \) and \( \|G\|_{L^2(dt)L^3(dx)} \) in the right hand side of the equation. Thus, if \( \Box \phi = F \) with Cauchy data \( f_0, f_1 \) and \( \Box \psi = g \) with Cauchy data \( g_0, g_1 \) Theorems 2 and 2.2 should read

\[
\int_0^T \int_{\mathbb{R}^3} |DQ(\phi, \psi)|^2 dx dt \leq c \left( \|f_0\|_{H^2(\mathbb{R}^3)} + \|f_1\|_{H^1(\mathbb{R}^3)} + \int_0^T \|\nabla F(t, \cdot)\|_{L^2(\mathbb{R}^3)} dt + \|F\|_{L^2([0,T])L^3(dx)} \right)^2 \times \left( \|g_0\|_{H^2(\mathbb{R}^3)} + \|g_1\|_{H^1(\mathbb{R}^3)} + \int_0^T \|\nabla G(t, \cdot)\|_{L^2(\mathbb{R}^3)} dt + \|G\|_{L^2([0,T])L^3(dx)} \right)^2
\]

The theorems are still true, as stated, for the null forms \( Q_{ij} \).

The problem is with the time derivative for the formula in the middle of page 1237. This has to be modified (for \( Q_0 \)) to

\[
\partial_t Q_0(\phi, \psi)(t, \cdot) = F(t, \cdot) \partial_t \psi(t, \cdot) + G(t, \cdot) \partial_t \phi(t, \cdot) + \int_0^t \int_0^t \partial_t Q_0(R(t - \tau)F(\tau, \cdot), R(t - \sigma)G(\sigma, \cdot)) d\tau d\sigma
\]

This was discovered by Lili He.

The originally stated estimate (without the terms \( \|F\|_{L^2(dt)L^3(dx)} \), \( \|G\|_{L^2(dt)L^3(dx)} \)) is true for the last term. However, the estimate is not true for the first two terms. For instance, if \( f_0 = 0 \), \( f_1 = 0 \),

\[
\|F\partial_t \psi\|_{L^2([0,T] \times \mathbb{R}^3)} \leq c \|F\|_{L^1([0,T]H^1(\mathbb{R}^3))} \times \left( \|g_0\|_{H^2(\mathbb{R}^3)} + \|g_1\|_{H^1(\mathbb{R}^3)} + \int_0^T \|G(t, \cdot)\|_{H^1(\mathbb{R}^3)} dt \right)
\]
cannot be true. For this reason, we dominate
\[ \| F \partial_t \psi \|_{L^2([0,T] \times \mathbb{R}^3)} \leq \| F \|_{L^2([0,T]) L^3(\mathbb{R}^3)} \| \nabla \partial_t \psi \|_{L^\infty([0,T] L^2(\mathbb{R}^3))} \]
\[ \leq \| F \|_{L^2([0,T]) L^3(\mathbb{R}^3)} \left( \| g_0 \|_{H^2(\mathbb{R}^3)} + \| g_1 \|_{H^1(\mathbb{R}^3)} + \int_0^T \| \nabla G(t, \cdot) \|_{L^2(\mathbb{R}^3)} dt \right) \]

The statement of our main theorem (Theorem 1) is not affected by this modification. The proof of Theorem 1 requires only a minor change.

Recall our original definitions of \( X_1, X_2, E_2 \),
\[ X_1^2(t) = \int_0^t \| DQ(\phi, \phi - \psi) \|_{L^2} d\tau \]
\[ X_2^2(t) = \int_0^t \| DQ(\psi, \phi - \psi) \|_{L^2} d\tau \]
\[ E_2(\phi)(t) = \sum_{0 \leq |A| \leq 2} \| D^A \phi(t, \cdot) \|_{L^2(\mathbb{R}^3)} \]

To estimate the newly introduced terms, we use Holder’s inequality and the Sobolev estimate to get
\[ \| Q(\phi, \phi - \psi)(t, \cdot) \|_{L^3(\mathbb{R}^3)} \leq C E_2(\phi)(t) E_2(\phi - \psi)(t) \]
thus, for solutions of the equation (3.1),
\[ \| \Box (\phi - \psi)(t, \cdot) \|_{L^3(\mathbb{R}^3)} \leq C E_2(\phi)(t) E_2(\phi - \psi)(t) \]
and
\[ \| \Box (\phi - \psi) \|_{L^2([0,T] L^3(\mathbb{R}^3))} \leq C \left( \int_0^t E_2^2(\phi - \psi)(\tau) d\tau \right)^{\frac{1}{2}} \]
Thus, when we apply the modified version of Theorem 2.2 to \( \phi \) and \( \phi - \psi \), we have the modification of (3.12)
\[ X_1(t) \leq C \int_0^t \| \nabla \Box (\phi - \psi)(\tau, \cdot) \|_{L^2} d\tau + \left( \int_0^t E_2^2(\phi - \psi)(\tau) d\tau \right)^{\frac{1}{2}} \]
which does not affect (3.8), the main estimate on which the uniqueness proof is based.

It is also possible to modify the statement and proof of Theorem 1 by using space-time norms involving only space derivatives.

Similar modifications apply to the existence part.

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