SOLUTIONS – MATH 130 – EXAM 2 – FALL 2014 – BOYLE

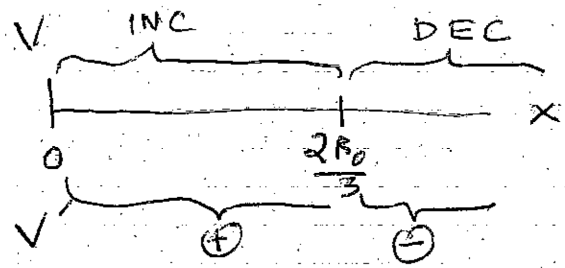
(1a) $V = C(R_0 - R)R^2$

$= CR_0R^2 - CR^3$

$\frac{dV}{dR} = 2CR_0R - 3CR^2$

$= CR(2R_0 - 3R)$

$\frac{dV}{dR} = 0$ at $R = \frac{2R_0}{3}$



$\boxed{V \text{ is } \underline{\text{MAX}} \text{ at } R = \frac{2R_0}{3}}$

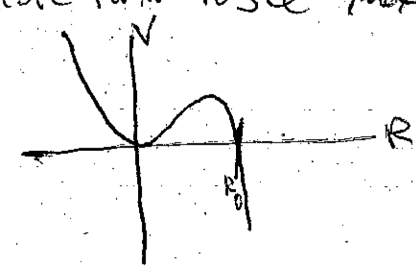
↖ You can also check for a local max or min with the First Derivative Test (the diagram here basically explains that test and its application to V) or the Second Derivative Test.

(1b)

$f(x) = \ln(6x^2 - 5x)$

$\boxed{f'(x) = \frac{12x - 5}{6x^2 - 5x}}$

For V in 1a, you can also just notice its cubic form to see "max":



2 (a) $f(x) = \sqrt{x^2+9} = (x^2+9)^{1/2}$

$f'(x) = \frac{1}{2}(x^2+9)^{-1/2}(2x)$

$= \frac{1}{2} \frac{2x}{\sqrt{x^2+9}}$

$f'(4) = \frac{1}{2}\frac{8}{5} = \frac{8}{10} = \boxed{\frac{4}{5}}$ or $\boxed{.8}$

tangent line equation:

$y - f(4) = f'(4)(x-4)$

$\cancel{\boxed{y-5 = \frac{8}{10}(x-4)}}$ ~~or~~ $\boxed{y-5 = \frac{4}{5}(x-4)}$

(b) $f(x) = \ln\sqrt{x+4}$

$= \ln(x+4)^{1/2}$

$= \frac{1}{2}\ln(x+4)$

$\boxed{f'(x) = \frac{1}{2}\frac{1}{x+4}}$

③ (a) $y = 3^{5x} = \left(e^{\ln 3}\right)^{5x}$

$y = e^{(\ln 3)5x} = e^{u}$

$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

$= e^{u}(5\ln 3)$

$$\boxed{y' = (5\ln 3)\, 3^{5x}}$$

(b) $y = \log_{10}(1-x)$

$y = \frac{1}{\ln 10}\ln(1-x)$

$$\boxed{y' = \left(\frac{1}{\ln 10}\right)\frac{-1}{1-x}}$$

4 (a) $y = e^{x^2} \cos x$

$y' = (e^{x^2})'(\cos x) + (e^{x^2})(\cos x)'$

$$\boxed{y' = 2xe^{x^2} \cos x + e^{x^2}(-\sin x)}$$

(b) $y = \sin(3x+2)$

$$\boxed{\frac{dy}{dx} = 3\cos(3x+2)}$$

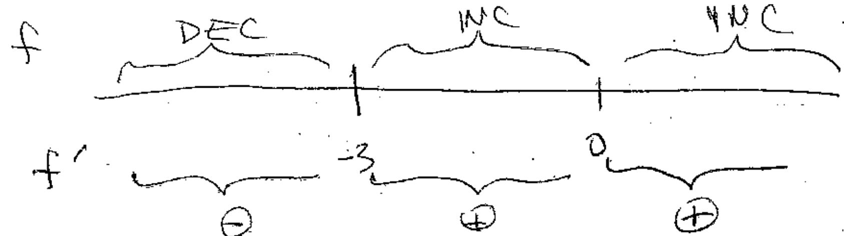
5(a) $f(x) = x^3 e^x$

$$f'(x) = 3x^2 e^x + x^3 e^x$$

$$0 = 3x^2 e^x + x^3 e^x$$

$$= x^2 e^x (3 + x)$$

$f'(x) = 0$ at $x = 0$ and $x = -3$



$\boxed{\text{f has a local minimum at } x = -3}$

(b) $f(x) = 2x - 9x^{2/3}$

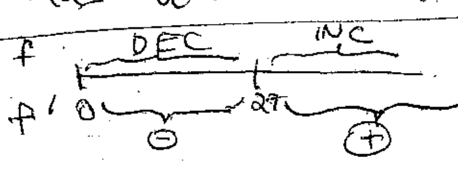
$$f'(x) = 2 - \left(\tfrac{2}{3}\right) 9x^{-1/3}$$

$$= 2 - \frac{6}{\sqrt[3]{x}}$$

$f'(x) = 0$ where $\sqrt[3]{x} = 3$, at $x = 27$

$\boxed{\text{f has a local minimum at } x = 27}$

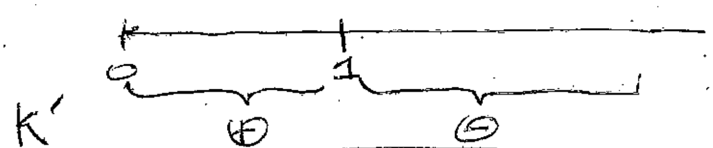
$\boxed{\text{f has a local max at } x = 0}$



(6) (a) $K(t) = \dfrac{5t}{t^2+1}$

$$K'(t) = \frac{(t^2+1)5 - (5t)(2t)}{(t^2+1)^2}$$

$$= \frac{5-5t^2}{(t^2+1)^2} = \frac{5(1-t^2)}{(t^2+1)^2}$$



K is increasing on [0, 1]
K is decreasing on [1, ∞)

(b) $y = (\cos x)^7$

$$y' = 7(\cos x)^6(-\sin x)$$

7 $f(x) = 2x + \frac{8}{x}$.

(a) asymptotes: $x = 0$

$y = 2x$

(b) $f'(x) = 2 - \frac{8}{x^2}$

$f'(x) = 0$ at $x = 2$ and $x = -2$

		-2		0		2	
f	INC		DEC		DEC		INC
f'	+		−		−		+

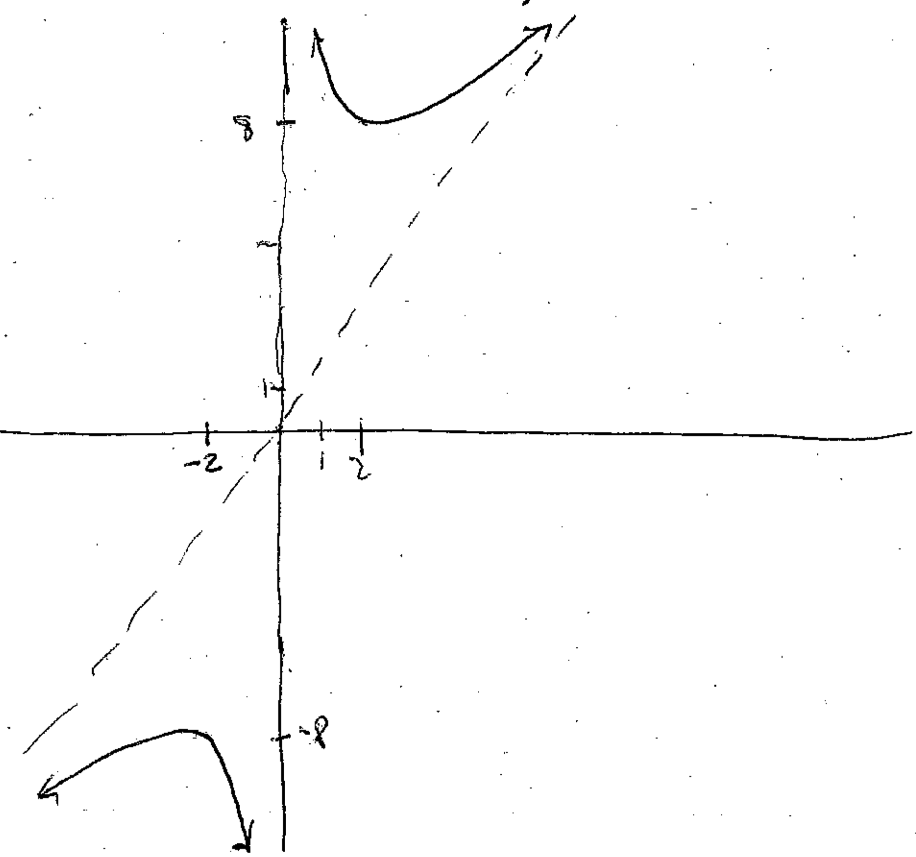
(c) $f''(x) = \frac{16}{x^3}$

graph of f is concave up on $(0, +\infty)$

" " " " " down " $(-\infty, 0)$

(d)

(graph is symmetric w.r.t. reflection thru origin)

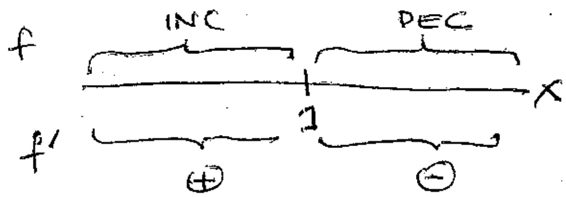


8 $f(x) = xe^{-x}$

(a) $y = 0$ is horiz. asymptote $\left(\lim_{x \to +\infty} f(x) = 0 \right)$

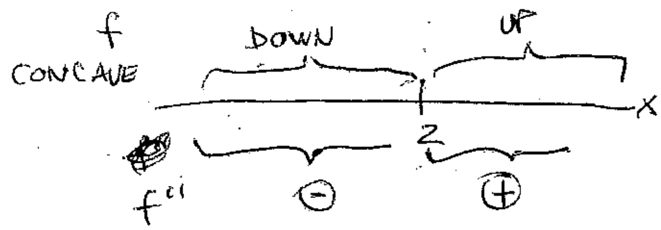
(b) $f' = e^{-x} - xe^{-x}$

$= e^{-x}(1-x)$

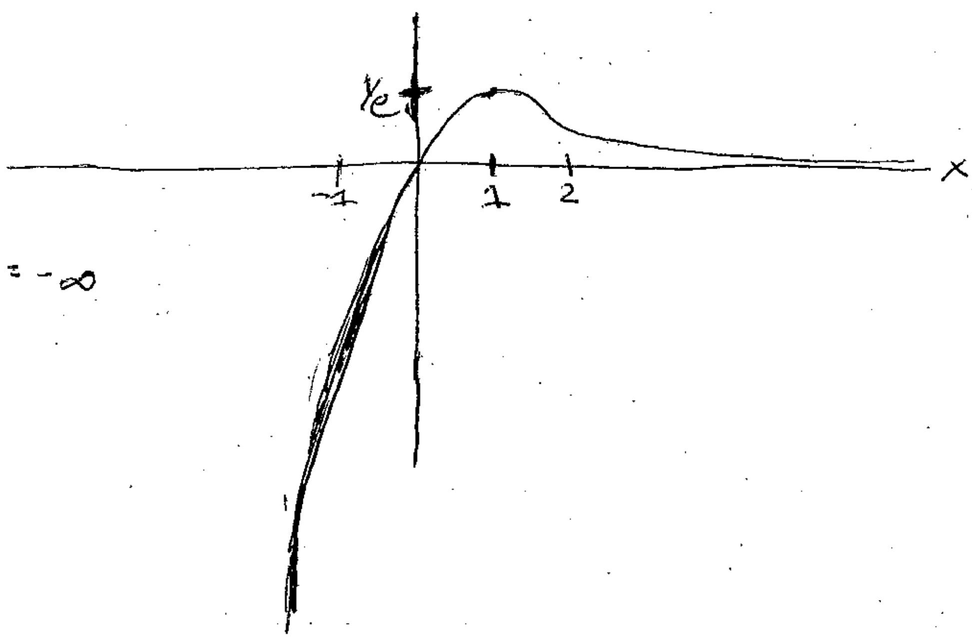


(c) $f'' = -e^{-x} - e^{-x} + xe^{-x}$

$= e^{-x}(x-2)$



(d)



$\lim_{x \to -\infty} f(x) = -\infty$