MATH 130 **Worksheet13 (11/26/2014)**

Student Name:

Section:

1. According to Fick's law, the diffusion of a solute across a cell membrane is given by

$$\boldsymbol{c'(t) = \frac{kA}{V}\,[C - c(t)]} \qquad (1)$$

where A is the area of the cell membrane, V is the volume of the cell, c(t) is the concentration inside the cell at time t, C is the concentration outside the cell. Here C, k and A are positive constants. From the constraint (1) on how c changes, we will derive a formula for c(t).

a) Given constants u_0 and b, compute the derivative u'(t) of the function $u(t) = u_0 e^{bt}$ and show that u'(t)=bu(t). (In fact, if u'(t)= bu(t), then u(t) must have this form.) One can evaluate u_0 by setting t=0, to get: u_0=u(0)

b) Let u(t) = C - c(t). From (1), write a new differential equation in terms of u and u'.

c) Solve that new differential equation to find a formula for u(t).

d) From this formula for u(t), derive a formula for c(t). We assume that c(t) is defined for t in some interval containing 0. (It can be shown that every solution of (1) has this form.)

2. The graph below shows the rate of inhalation of oxygen (in liters per minute) by a person riding a bicycle very rapidly for 10 minutes. Estimate the total volume of oxygen inhaled in the first 20 minutes after the beginning of the ride. Use rectangles with widths of 1 minute.