MATH 130
Solutions to Worksheet13 (11/26/2014)

1. a)

$$u(t) = u_0\, e^{bt}$$

$$u'(t) = b(u_0\, e^{bt})$$

We will show now that any function w on an interval which satisfies $w'(t) = b\,w(t)$ must have the form $w(t) = w_0\, e^{bt}$ for some constant w_0. (This is a bonus for you; it was not part of the original exercise).

Given $w'(t) = bw$(t). Let $f(t) = {}^{w(t)}/_{e^{bt}}$ (well defined since u is never zero). Differentiate with the quotient rule to get:

$$f' = \frac{uw' - u'w}{e^{2bt}} = \frac{ubw - buw}{e^{2bt}} = 0$$

This f is constant on the interval since $f' = 0$ on the interval. The constant is the required w_0.

b) Letting u(t) = C - c(t), we first take the derivative of u(t):

$$u'(t) = -c'(t)$$

Rewriting the equation in terms of u(t), u'(t), we would have:

$$u'(t) = -\frac{kA}{V}\,[u(t)]$$

c) Solving the differential equation to obtain the anti-derivative of u'(t):

$$u(t) = \; u_0 . e^{-\frac{kA}{V}t}$$

d) Now we replace u(t) by [C- c(t)], and u_0 by (C – c_0) where c_0 is c(0):

$$C - c(t) = \; (C - c_0). e^{-\frac{kA}{V}t}$$

$$c(t) = C + (c_0 - C). e^{-\frac{kA}{V}t}$$

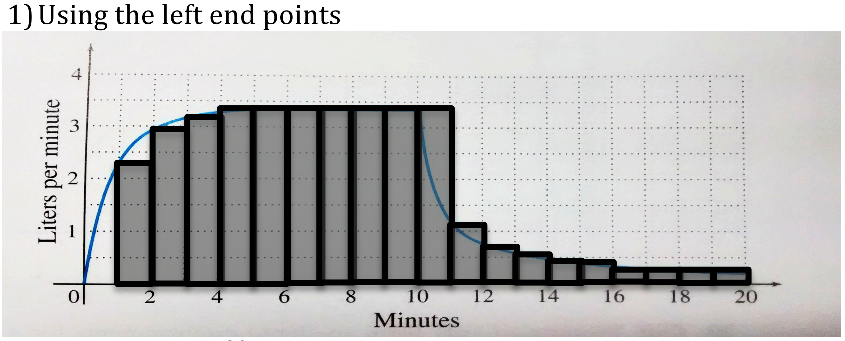
Having a positive constant k means that the second term (the difference between the concentration inside and outside of the membrane) is decaying and the concentration inside the cell is getting closer to the concentration of outside.

2. The graph below shows the rate of inhalation of oxygen (in liters per minute) by a person riding a bicycle very rapidly for 10 minutes. Estimate the total volume of oxygen inhaled in the first 20 minutes after the beginning of the ride. Use rectangles with widths of 1 minute. (Use the left end points, then the right endpoints, then give the average of those answers.)

$\Delta x = 1\ min$

x(min)	f(x)(L/min)	x(min)	f(x)(L/min)
0	0		
1	2.3	11	1.2
2	2.9	12	0.8
3	3.2	13	0.6
4	3.3	14	0.5
5	3.3	15	0.4
6	3.3	16	0.3
7	3.3	17	0.3
8	3.3	18	0.2
9	3.3	19	0.2
10	3.3	20	0.2

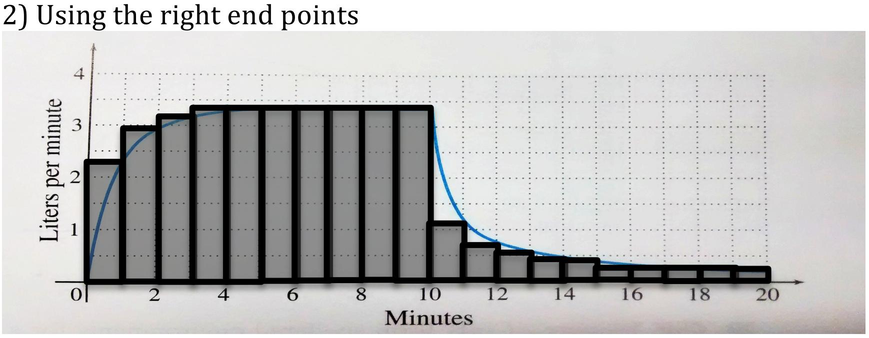
1) Using the left end points



$$Volume = \sum_{i=1}^{20} f(xi)\Delta x$$

$$= 0\times1 + 2.3\times1 + 2.9\times1 + 3.2\times1 + 3.3\times1 + 3.3\times1 + 3.3\times1 + 3.3\times1 + 3.3\times1 + 3.3\times1 + 3.3\times1 + 1.2\times1 + 0.8\times1 + 0.6\times1 + 0.5\times1 + 0.4\times1 + 0.3\times1 + 0.3\times1 + 0.2\times1 + 0.2\times1 = 36\ L$$

2) Using the right end points



$$Volume = \sum_{i=1}^{20} f(xi)\Delta x$$

$$= 2.3\times1 + 2.9\times1 + 3.2\times1 + 3.3\times1 + 3.3\times1 + 3.3\times1 + 3.3\times1 + 3.3\times1 + 3.3\times1 + 3.3\times1 + 1.2\times1 + 0.8\times1 + 0.6\times1 + 0.5\times1 + 0.4\times1 + 0.3\times1 + 0.3\times1 + 0.2\times1 + 0.2\times1 + 0.2\times1 = 36.2\ L$$

Average of Volumes = 36.1 L