

Math 130 – Spring 2015 – Boyle –Exam 1

- NO CALCULATORS OR ELECTRONIC DEVICES ALLOWED.
- Where a calculator would be used, give your answer as an expression a calculator could evaluate.
- Use a separate answer sheet for each of the seven questions.
- When you are asked to find a limit, the answer could be a number; ∞ ; $-\infty$; or DNE. (DNE = “does not exist”, including the case “is not defined”.)
- Give your pledge on page 1 only, covering the whole test.
- Draw a box around a final answer to a problem.

1. (13 points)

- (4 pts) What is the domain of $y = \ln(x - 6)$? What is the range?
- (3 pts) Solve $\log_2(x) = -3$.
- (6 pts) Solve $\sqrt{e^x} = e^x e^{x+1}$.

2. (10 points)

Potassium-40, with a half-life of 1.25 billion years, has been used by geochronologists trying to sort out the mass extinction of 250 million years ago. What fraction of the Potassium-40 remains from a creature that died 250 million years ago?

3. (10 points)

- (4 pts) Find all values of x between 0 and 2π for which $\sin x = 1/2$.
- (3 pts) What is the period of the function $y = 5 \sin(3t + 2)$?
- (3 pts) For the population model function

$$P(t) = \frac{8}{1 + 4e^{-3t}}$$

compute $\lim_{t \rightarrow \infty} P(t)$.

4. (12 points) Let position be measured in feet and let time be measured in seconds. Suppose the position of an object moving in a straight line is given by $s(t) = |t|$.

- (6 pts) What is the average velocity between $t = -3$ and $t = 1$?
- (3 pts) What is the instantaneous velocity at $t = 2$?
- (3 pts) What is the instantaneous velocity at $t = -2$?

5. (16 points) Determine the following limits. (4 points each)

(a) $\lim_{x \rightarrow 0} f(x)$ with f defined by $f(x) = \begin{cases} 3x + 1 & \text{if } x \neq 0 \\ 2 & \text{if } x = 0 \end{cases}$

(b) $\lim_{x \rightarrow -1} \frac{x^2 + 4x + 3}{x + 1}$

(c) $\lim_{x \rightarrow \frac{\pi}{2}^-} \tan(x)$

(d) $\lim_{x \rightarrow -\infty} \frac{3x^5 + 5x^4 - 19,000}{5x^5 + 4x^4 + 3x^3}$

6. (12 points) (a) (8 pts) A tumor is approximately spherical in shape. If the radius of the tumor grows from 13 mm to 15 mm, what is the linear approximation to the change in the volume of the tumor?

(b) (4 pts) Given an example of a continuous function $y = f(x)$, from $(-\infty, \infty)$ to $(-\infty, \infty)$, and a number a such that $f'(a)$ does not exist. You do not have to give a proof.

7. (17 points) (a) (9 pts) Find an equation for the tangent line of the graph of $y = \sqrt{x}$ at the point (9, 3).

Determine the following (2 pts each).

(b) $\lim_{x \rightarrow +\infty} \frac{\sin(x)}{x}$

(d) $\lim_{x \rightarrow +\infty} \frac{x^5}{e^x}$

(c) $\lim_{x \rightarrow +\infty} \sin(x)$

(e) $\lim_{x \rightarrow +\infty} \frac{\sqrt{x}}{\ln(x)}$