Math 130 – Spring 2015 – Boyle –Exam 2

- NO CALCULATORS OR ELECTRONIC DEVICES ALLOWED.
- Where a calculator would be used, give your answer as an expression a calculator could evaluate. For full credit, simplify expressions appropriately.
- Use a separate answer sheet for each of the SEVEN questions.
- Give your pledge on page 1 only, covering the whole test.
- Draw a box around a final answer to a problem.

1. (14 points)
For each of the following functions, find the formula for y'.
(a) (7 pts) $y = 2^{-5x}$.
(b) (7 pts) $y = \ln(|\sin(2x)|)$.

2. (14 points)
(a) (7 pts) Given $y = \log_{10}(\sqrt{3x})$, find the formula for y'.
(b) (7 pts) Given $y = (\cos(x))/(x^2 + 1)$, find the formula for y'.

3. (14 points)
Find every relative extreme value of the function $f(x) = (\ln x)(x^2)$, and indicate which are relative maxima and which are relative minima. (Remember, values are outputs.)

4. (14 points)
For each of the following functions, determine all asymptotes; if there is no asymptote for a function, say so.

$$f(x) = 7x + \frac{\cos x}{x} \qquad g(x) = \frac{8x^4 + 3x + 1}{x^2 + 5} \qquad h(x) = \ln(x) \ .$$

5. (14 points) (4 pts)
The formulas for the volume V and surface area A of a ball as a function of its radius R are $V = \frac{4}{3}\pi R^3$ and $A = 4\pi R^2$. There are numbers C and s such that $A = CV^s$ gives the area of a ball as a function of its volume.

a. (2 pts) What is the relationship between A and dV/dR?

b. (4 pts) What is s?

c. (4 pts) Find the formula which gives dA/dV as a function of V. (You do not have to solve for C, but you must use the correct number for s.)

d. (4 points) Compute $\lim_{R \to \infty} dA/dR$ and $\lim_{V \to \infty} dA/dV$.

6. (14 points) For a given positive constant r, the Ricker model of population uses the function $P(x) = xe^{r(1-x)}$ to estimate the population one year from today, given that the population now (in suitable units) is x. The domain of P is $[0, \infty)$.

(a) (2 pts) Find all asymptotes for P (if there are none, say so).

(b) (4 pts) Find the intervals on which f is increasing/decreasing.

(c) (2 pts) Determine all inputs x at which f has a relative maximum or minimum (say which).

(d) (2 pts) You may assume $P''(x) = (e^{r(1-x)})(-2r + r^2x)$. Find the intervals on which the graph of f is concave up/down.

(e) (4 points) For the parameter value $r = 1$, graph f.

7. (16 points)

(a) (4 pts) You are given the following table of values:

x	1	2	3	4
$f(x)$	2	4	1	3
$f'(x)$	-6	-7	-8	-9
$g(x)$	2	3	4	1
$g'(x)$	2/7	3/7	4/7	5/7

If $h(x) = g(f(x))$, what is $h'(1)$?

(b) Answer each of the following TRUE or FALSE. No proof required.

(i) (4 pts) The largest number of local extreme values a polynomial of degree 5 can have is 5.

(ii) (4 pts) If f is the function $f(x) = e^x$, then $f'(2x) = 2f'(x)$, for every x.

(iii) (4 pts) In the Fitz-Hugh-Nagumo model of neuron communication, the rate of change of the electrical potential with respect to time is given as a function of the potential v by $f(v) = v(a - v)(v - 1)$. Suppose $a = 1/4$.

True or False: The electrical potential is increasing with respect to time when $v = 1/5$.